Fiscal Insurance and the Pricing of Government Debt[1]

Filippo Cavaleri
University of Chicago

November 18, 2025
Latest version

Frequently Updated – Please Check for the Latest Version

Abstract

I model the impact of government debt on bond prices when fiscal policy interacts with uninsurable risks. Debt levels impact asset valuations through future tax adjustments that alter households' risks and, hence, their pricing kernel. Households' exposure to uninsurable risks scales with debt levels, but the relation is non-monotonic if tax adjustments amplify redistributive risks. State-contingent debt can provide better insurance than risk-free debt by shifting the distribution of income dispersion across aggregate states. The covariance between taxes and pricing kernels identifies whether further debt issuance improves risk sharing. Debt issuance erodes fiscal insurance and hurts welfare beyond a debt threshold. This debt threshold declines with the level of foreign demand and increases with the progressivity of taxation. Foreign demand weakens fiscal insurance but crowds in domestic bond demand if sufficiently inelastic. In a dynamic setting, fiscal insurance raises interest rates, compresses risk premia, and reduces investment. I show that even in incomplete markets, a version of Ricardian neutrality holds if taxes are lump-sum. My results indicate that debt alone does not improve risk sharing and that supply effects will persist even if government bonds lose their specialness.

Keywords: Fiscal policy; Incomplete markets; Risk sharing; Heterogeneous agents economies.
JEL Classification: G12, G18, H31, H63

[1]First draft: October 2025. I thank Ben Brooks, Lars Hansen, Zhiguo He (co-chair), Stefan Nagel (co-chair) and Quentin Vandeweyer for their invaluable guidance and generous support at all stages of this project. For comments and suggestions I thank Federico Baldi-Lanfranchi, Jaroslav Borovicka (discussant), Aditya Chaudhry, Doug Diamond, Julie Z. Fu, Mike Golosov, Kilian Huber, Greg Kaplan, Anil Kashyap, Rohan Kekre, Paymon Khorrami, Ralph Koijen, Guido Lorenzoni, Federico Mainardi, Lira Mota, Scott Nelson, Simon Oh, Carolin Pflueger, Raghuram Rajan, Angelo Ranaldo, Taisiya Sikorskaya, Dejanir Silva, and Eric Zwick. All errors are my own.

1 Introduction

Government bonds and taxes are central elements of households' balance sheets, although they have distinct risk characteristics. Government bonds promise the same return to all holders, while tax liabilities depend on individual circumstances such as job loss or income fluctuations. This combination creates uncertainty about households' fiscal incidence that is often uninsurable: who ultimately will bear the burden of debt? Typically, more affluent households pay more taxes (Heathcote, Storesletten, & Violante, 2017), so the debt burden falls on the more fortunate.[1] Hence, holding government bonds against an equivalent share of future tax liabilities offers a form of insurance. In contrast to representative agent models, such *fiscal insurance* implies that government debt is net wealth (Barro, 1974).

This paper develops a tractable general equilibrium framework in which fiscal policy interacts with uninsurable risks. The main message is that debt and taxes *together* determine households' exposure to uninsurable risks and how this exposure varies over time and across states. As a result, fiscal policy impacts households' *individual* pricing kernels and asset valuations, including the valuation of government debt itself. My goal is to examine the implications of fiscal insurance for the equilibrium pricing of government debt.[2] Specifically, how does fiscal insurance affect bond yields and expected returns? When does government debt cease to provide insurance? How do these effects depend on the level of debt and the structure of taxation?[3] Answering these questions is particularly important today because fiscal risks increase with debt levels and have ramifications for fiscal capacity.

The impact of fiscal policy on bond yields and expected returns depends critically on which households bear the debt burden. Fiscal insurance shifts the burden to the more fortunate: households pay more taxes after favorable income shocks. Taxes reflect *idiosyncratic* risks, and households discount them at a higher rate than debt: the net value of government debt is positive. The government budget constraint links this net value to the quantity of debt outstanding: households' exposure to uninsurable risk scales with debt levels. This is because debt supply influences future taxes, affecting households' equilibrium exposure to uninsurable risk and hence *individual* pricing kernels. The covariance between individual pricing kernels and tax burdens determines whether the net value of government debt is positive. Using individual pricing kernels is revealing: for a representative agent consuming the entire endowment, the relative tax burden is constant and fiscal insurance has no value.

While private and public assets are perfect substitutes and trade at the same price, government debt

[1] Earlier contributions by Chan (1983) and Kimball and Mankiw (1989) show that changes in the timing of procyclical income taxes have substantial impacts on consumption and savings. Heathcote et al. (2017) show that insurance is an important determinant of the optimal progressivity of taxation.

[2] A growing body of empirical evidence suggests that the size and composition of government debt influence Treasury yields and many other asset classes. Treasury supply affects liquidity and safety premia (Greenwood, Hanson, & Stein, 2015; Krishnamurthy & Li, 2022; Krishnamurthy & Vissing-Jorgensen, 2012; Nagel, 2016), the maturity choice of the corporate sector (Greenwood, Hanson, & Stein, 2010), and bond risk premia (D'Amico & King, 2013; Greenwood & Vayanos, 2014).

[3] In standard asset pricing models the answers are trivial. How a government finances its spending is irrelevant for asset prices. Debt is backed by taxes that households pay, so households' consumption is independent of the level of debt.

is distinct. An increase in public debt triggers future tax adjustments that affect all households, whereas private debt does not. Crucially, these tax adjustments change households' individual pricing kernels, and thus asset prices. First, an increase in government debt leads to higher future tax rates that affect households' exposure to uninsurable risks. Second, debt issuance changes the composition of households' portfolios and thus the share of income they earn through capital gains and interest payments. The takeaway is that movements in the supply of a given asset can lead to different economic outcomes depending on who the issuer is and how interest repayments are backed.

I start with a two-period economy in which a government issues debt and collects taxes to finance an exogenous stream of purchases. Households face aggregate and idiosyncratic risks, but they can only trade a complete set of claims on aggregate shocks subject to natural borrowing limits. In contrast, households cannot write contracts on idiosyncratic risks: households are ex ante identical, but ex post heterogeneous.[4] Because it provides a good approximation of the US tax system, I consider an affine tax structure: the government raises revenue from output and lump-sum taxes. Future tax rates adjust according to a given rule that pins down the revenue share by each tax instrument.[5]

Households' fiscal incidence depends on idiosyncratic shocks, and a key innovation of the paper is to explicitly model the uncertainty over which households bear the debt burden. This fiscal-incidence risk is distinct from income risk: rising public debt exposes households to uninsurable uncertainty about future tax liabilities that does not necessarily move with their income. To capture this idea, I introduce a *stochastic* lump-sum tax in which each household pays a random share of aggregate lump-sum taxes. A standard lump-sum tax is the special case where the burden is identical across households. While output taxes provide partial insurance by falling more heavily on households with high income realizations, stochastic lump-sum taxes generate fiscal-incidence risk and make taxation less progressive. This distinction between income risk and fiscal-incidence risk is crucial because the extent to which government debt provides insurance depends on the tax structure.

Fiscal insurance implies that the relationship between the risk-free rate and the quantity of government debt is non-monotonic. There are two regions that differ in how asset prices respond to additional debt. At low levels of debt, higher issuance raises the real risk-free rate. At high levels of debt, however, government debt ceases to provide insurance, so that further issuance lowers the risk-free rate. The boundary between these two regions defines a debt threshold at which government debt ceases to provide insurance. This threshold depends on tax progressivity and on the relative volatility of income shocks versus fiscal risk. Specifically, the more the relative tax burden is decoupled from households' income shocks, the lower this threshold is.[6]

Interestingly, an increase in risk-free debt also affects the price of aggregate risk. When the gov-

[4]The closest references are Angeletos (2007), Di Tella (2020), and Brunnermeier, Merkel, and Sannikov (2024). I emphasize fiscal insurance as a driver of risk sharing and study its implications for interest rates and risk premia.

[5]The model takes the government's behavior as given. My emphasis is thus on the positive implications of debt issuance for future taxation rather than normative prescriptions about optimal taxes.

[6]The response of bond yields to changes in the stock of public debt is in sharp contrast to partial equilibrium models that attribute the entire response to bond risk premia (Greenwood & Vayanos, 2014; Haddad, Moreira, & Muir, 2025).

ernment issues no debt, there is no trade in assets: precautionary savings depress the risk-free rate relative to a representative-agent economy with the same endowment, yet the aggregate risk premium remains the same (Krueger & Lustig, 2010). Once the government borrows, the bond market opens and households hold financial claims, so a larger share of their income now comes from portfolio returns rather than from the risky endowment. I show that with pure lump-sum taxes, the ratio of aggregate state prices is independent of debt supply. But when taxes interact with uninsurable risk, this interaction affects how consumption dispersion varies across aggregate states. As a result, the ratio of state prices depends on fiscal policy. In particular, government debt issuance can dampen the countercyclical variation in idiosyncratic consumption risk. This effect is even stronger if the government issues negative-beta assets that provide greater insurance in bad states of the world.

I characterize the net value of government debt through the covariance between tax burdens and individual pricing kernels. If tax burdens increase after favorable idiosyncratic shocks, taxes become risky liabilities and households discount them at a higher rate than returns on government debt. This difference in discount rates creates a valuation gap in individual budget constraints that reflects households' shadow prices of uninsurable risks. Importantly, this gap does not wash out after aggregating across all households. Thus, fiscal insurance implies that the market value of government debt exceeds the aggregate present value of individual taxes: households' perceived wealth is higher, and aggregate welfare improves.[7] Absent distortionary taxes that affect aggregate output, fiscal policy has real effects only by altering the allocation of risk in the economy. This covariance determines whether households' exposure to uninsurable risks varies with the level of debt. In representative agent models, a single household pays all taxes, so the relative tax burden is constant, and the market value of government debt always equals the present value of taxes.

I show that a version of Ricardian equivalence holds under incomplete markets in the special case of pure lump-sum taxes, where each household pays a fixed, non-random share of aggregate taxes. In this case, the timing of taxes does not interact with households' exposure to uninsurable risks, so the value of fiscal insurance is zero and the equilibrium is independent of the stock of public debt. Households can offset lump-sum taxes through private asset holdings: precautionary demand for bonds lowers the risk-free rate relative to a representative-agent economy, but the constant tax burden is discounted at the same rate as government debt. As a result, government debt is not net wealth, and an increase in debt shifts asset demand in a way that exactly offsets any effect on yields. This example demonstrates that taxes and debt *together* generate insurance.

This mechanism delivers two important lessons for how the stock of public debt affects individual pricing kernels. First, government debt need not be special to have real effects. Second, demand curves for both private and public assets are not invariant to fiscal policy. As a result, the level of the risk-free rate and aggregate risk premia depend on the stock of public debt and the structure of taxation. My fiscal perspective differs conceptually from and is complementary to other explanations that rely on bond

[7] This mechanism is distinct from the buffer stock idea that Aiyagari and McGrattan (1998) describe. I show that government debt has no real effects if taxation is lump-sum and there is no uncertainty about its distribution.

characteristics such as convenience yields (Krishnamurthy & Vissing-Jorgensen, 2012), service flows from retrading (Brunnermeier et al., 2024), or portfolio-balance effects (Greenwood & Vayanos, 2014).[8]

Without debt, a no-trade equilibrium emerges and the economy operates as if financial markets did not exist (see Constantinides and Duffie (1996)). Fiscal insurance can take households out of autarky, but positive wealth contributions do not necessarily translate into higher welfare (Barsky, Mankiw, & Zeldes, 1986). To understand the welfare implications, I consider a benevolent government that takes the tax structure as given: the government would choose a debt level such that individual pricing kernels are orthogonal to fiscal incidence. Intuitively, households can insure against aggregate risk through financial markets, so government debt should be used primarily to help households hedge idiosyncratic risk. When marginal utilities and tax burdens are positively correlated, the government could reduce consumption volatility by issuing more debt. The optimum occurs when this correlation is zero: at this point, the government has exhausted its role to provide insurance. Higher borrowing costs may signal that debt provides insurance to households, whereas introducing additional uncertainty in taxation could reduce borrowing costs but at the expense of welfare.

The second part of the paper extends the analysis to include foreign investors and nominal debt, two salient features of the current institutional setting. In an open economy, fiscal insurance comes at the expense of redistribution from domestic to foreign investors: domestic households face the same fiscal adjustment, but part of the bond income accrues abroad. I find that foreign demand reduces the insurance benefit of government debt, yet it is complementary to domestic demand. Wealth redistribution abroad lowers future domestic consumption and raises current savings, which reduces interest rates relative to an economy without foreign investors. These effects are attenuated if foreign demand is price elastic, as higher domestic demand raises bond prices and crowds out foreign investors.
The impact of nominal debt issuance crucially depends on both the distributional consequences of inflation and on whether inflation occurs in good or bad times. An increase in inflation lowers the real return on debt and reduces the required tax adjustment, so that the value of fiscal insurance declines. In particular, inflation that disproportionately affects households with lower income or wealth can be more regressive than taxes, further weakening the insurance provided by government debt.

In the third part of the paper, I extend the framework to continuous time with capital accumulation and capital risk. To preserve tractability, I assume that households are subject to uninsurable investment risk. As a result, consumption and saving decisions are linear in wealth and aggregate quantities do not depend on the wealth distribution. Continuous time delivers an interesting insight. If preferences are homothetic and taxation is proportional, the share of interest income and tax liabilities always coincide. Still, government debt has real effects through households' pricing kernels. The wealth share of capital is generally different than one because of the valuation gap between tax liabilities and government debt. This wealth share is economically important as it is the state variable that determines households' decisions and characterizes aggregate equilibrium quantities.

[8]The way in which this form of insurance works in general equilibrium is very different from the public provision of liquidity in Holmström and Tirole (1998). In Holmström and Tirole (1998), government debt has no real effects without aggregate uncertainty, but it provides liquidity under pure aggregate uncertainty, which is the opposite of this paper.

The dynamic framework reveals that fiscal insurance lowers the investment rate and thus the economy's growth rate. When the covariance between individual pricing kernels and tax burden is negative, all households hold government bonds they perceive as more valuable than their future tax liabilities, so the wealth share of capital falls below one. In representative agent economies, the tax burden is constant, so the value of government debt exactly offsets the present value of taxes and the wealth share of capital is one. With incomplete markets, however, fiscal insurance lowers the present value of individual tax obligations and households feel wealthier. Interest from government bonds replaces risky capital income, so households effectively hold less than 100% of their net wealth in capital. Households respond to this wealth effect by consuming more and investing less. Because this conclusion does not rely on the specific tax instrument, a key message is that redistributive taxation lowers investment without necessarily distorting the investment margin directly.

In the special case of pure lump-sum taxes, the wealth share of capital is always one. As a result, aggregate quantities are independent of fiscal policy. This result delivers an important conceptual point. It is common in the literature to interpret a store of value as a savings technology that offers absolute certainty of repayment and allows households to self-insure against idiosyncratic shocks. While this interpretation may apply to assets in positive net supply such as capital, gold, or money, government bonds are ultimately *financial claims*. For a financial claim to become a store of value, it must offer more than a safe payoff. Specifically, households experiencing a series of bad idiosyncratic shocks should bear less of the debt burden. Thus, the role of government debt as a store of value emerges from the tax structure. In general, to establish whether any financial claim can improve risk sharing, the actual return on that asset must account for who bears the liabilities backing it and when.

The net wealth contribution of government debt maps into the difference between two distinct valuation approaches. First, one can value the aggregate tax claim using a market stochastic discount factor $\overline{\xi}_t$. Ignoring government purchases, the intertemporal government budget constraint implies that the present value of aggregate tax revenue equals the market value of government debt (Jiang, Lustig, Van Nieuwerburgh, & Xiaolan, 2024). Second, one could also value each household's tax liabilities using their individual marginal utilities and then aggregate across all households. This valuation exercise is equivalent to discounting aggregate tax revenue with a tax-weighted stochastic discount factor ξ_t^{**}. The key differences is that $\overline{\xi}_t$ and ξ_t^{**} decay at different rates; my approach thus extends Brunnermeier et al. (2024) by showing that taxes are important in determining whether government debt improves risk sharing: valuations coincide if markets are complete or if taxes are pure lump-sum.

1.1 Related Literature and Contribution

This paper bridges asset pricing and public finance by studying how fiscal policy affects individual pricing kernels through its impact on households' exposure to aggregate and uninsurable risks. Understanding how fiscal policy shapes pricing kernels is important for assessing how fiscal policy impacts to the valuation of government debt, determines fiscal sustainability, and affects several other

asset prices. I show that debt and taxes together provide a form of fiscal insurance that neither alone can offer, and I characterize the implications for bond prices. In contrast to most of recent finance literature (e.g. Greenwood and Vayanos (2014)), I take a general equilibrium approach: the risk-free rate responds endogenously to debt issuance, generating feedback effects that are key to understand how fiscal policy feeds back into the valuation of government debt.
My paper speaks to a longstanding debate about whether government debt impacts interest rates and constitutes net wealth (Barro, 1974; Tobin, 1971). Earlier work typically examines the effects of taxes in isolation. While income taxes are known to have real effects in combination with uninsurable income risks (Barsky et al., 1986; Chan, 1983; Kimball & Mankiw, 1989), their implications for asset prices are less studied. My contribution is to explicitly model the uncertainty over which households will bear the tax burden and link it to debt levels. Specifically, the introduction of a stochastic lump-sum captures the realistic feature that uncertainty over debt burdens increases with the level of debt (Bianchi, Dabla-Norris, & Khalid, 2025) and generates a set of novel asset pricing predictions. I show that valuation differences between government debt and taxes reveal whether further debt issuance improves welfare and have real consequences in production economies where households have access to a storage technology; households feel wealthier, consume more, and invest less.

Building on earlier work by Bewley (1979), Huggett (1993), and Aiyagari and McGrattan (1998), a recent growing literature emphasizes the role of money (Di Tella, 2020) and government debt (Brunnermeier et al., 2024) in improving risk sharing. A key distinction between earlier and later work lies in the treatment of borrowing constraints. Early models assume households cannot borrow at all, so government debt relaxes borrowing constraints and has real effects (Scheinkman & Weiss, 1986). More recent work imposes natural borrowing limits and shows that debt enhances risk sharing through liquidity premia (Di Tella, 2020) or service flows (Brunnermeier et al., 2024). My work differs in two respects. First, I argue that the combination of debt and taxes generates fiscal insurance that neither alone can provide. The benchmark with pure lump-sum taxes demonstrates that debt by itself yields no risk-sharing benefit. Second, I show that the tax structure entirely determines whether an increase in government debt improves risk sharing.

A growing empirical asset pricing literature documents that Treasury supply affects bond yields and other asset prices (Greenwood et al., 2015; Greenwood & Vayanos, 2014; Krishnamurthy & Li, 2022; Krishnamurthy & Vissing-Jorgensen, 2012; Nagel, 2016). This evidence poses a challenge to standard asset pricing models in which Ricardian equivalence holds. Popular departures rely on intermediaries' limited risk bearing capacity (Greenwood & Vayanos, 2014; Haddad et al., 2025; Vayanos & Vila, 2021) or special demand for the safety and liquidity services that Treasuries provide (d'Avernas & Vandeweyer, 2024; Drechsler, Savov, & Schnabl, 2018; Krishnamurthy & Vissing-Jorgensen, 2012). A common theme is that government bonds have special features that differentiate it from private assets. I pursue a complementary approach in which government and private bonds are perfect substitutes and show that debt supply affects asset prices by impacting the equilibrium risk exposures. My notion of fiscal risk also differs from recent work that relies on hand-to-mouth households (d'Avernas, Hubert de Fraisse, Ning, & Vandeweyer, 2024; Gomez Cram, Kung, Lustig, & Zeke, 2025). Instead, I emphasize that households' exposure to uninsurable risks scale with the quantity of outstanding debt.

The infinite-horizon extension introduces aggregate risk and idiosyncratic investment risk, building on Angeletos (2007) and Brunnermeier et al. (2024). This differs from the HANK literature, which typically studies economies with idiosyncratic labor income risk and borrowing constraints but abstracts from aggregate risk (Auclert, Rognlie, & Straub, 2024; Kaplan, Nikolakoudis, & Violante, 2023). I characterize how both the risk-free rate and aggregate risk premia vary with debt supply and show that debt shocks affect households' exposure to idiosyncratic risk (Di Tella, 2017).
The link between individual pricing kernels and taxation relates to recent work on the valuation of government debt (Jiang, Lustig, Van Nieuwerburgh, & Xiaolan, 2023). In the infinite-horizon extension, nothing rules out rational bubbles on government debt (Brunnermeier et al., 2024). However, I demonstrate that both bubbles and fiscal insurance can improve risk sharing, but neither requires the other. Another feature of the model is that government debt opens trading in financial assets, whereas Constantinides and Duffie (1996) and Krueger and Lustig (2010) study no-trade equilibria.

Finally, my paper relates to a large literature on optimal taxation and debt management. The mainstream view is that government debt serves to smooth tax distortions over time (Aiyagari, Marcet, Sargent, & Seppälä, 2002; Bhandari, Evans, Golosov, & Sargent, 2017a; Lucas & Stokey, 1983). The literature on optimal taxation also studies the trade-off between efficiency and redistribution (Golosov, Kocherlakota, & Tsyvinski, 2003; Kocherlakota, 2005); but it often abstracts from government debt because the availability of lump-sum taxes males the mix between debt and lump-sum transfers is indeterminate (Werning, 2007). While fiscal policy in these frameworks directly affects equilibrium allocations and thus asset prices, this literature does not characterize the asset pricing implications. My contribution is to explicitly study how fiscal insurance affects equilibrium asset prices through individual pricing kernels, and how this feeds back into the valuation of government debt.

1.2 Organization

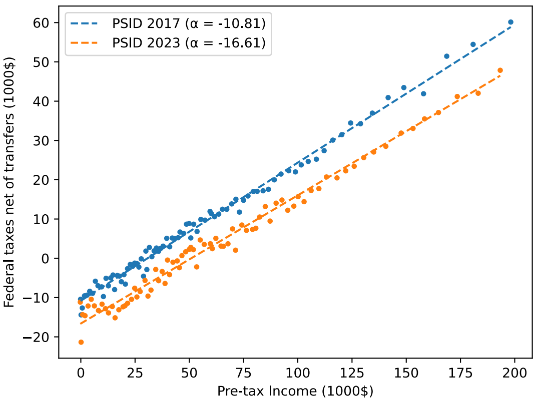
The rest of the paper is as follows. Section 2 presents motivating facts about individual taxation and the return on government debt. Section 3 describes the two-period model and derives the key results. Section 4 extends the analysis to foreign investor and nominal debt, and discusses robustness of the two-period framework. Section 5 extends the analysis to a production economy and describes how differences in valuation between taxes and debt have real effects. Section 6 concludes.

2 Motivating Facts and a Stylized Example

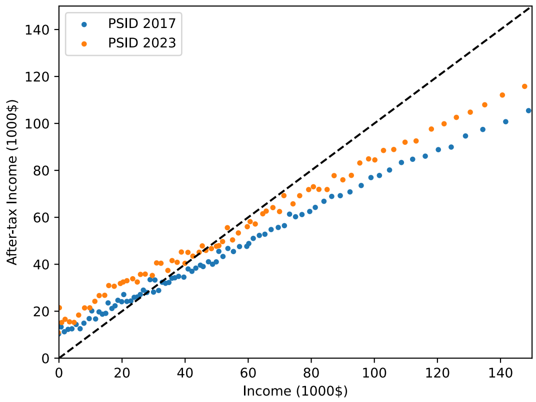
The purpose of this section is to provide empirical justification for the main mechanism of the model. I first show that an affine tax schedule provides a reasonable approximation of the US tax system, and that taxes compress the cross-sectional dispersion in incomes. Second, I present anecdotal evidence that relative tax burdens changes for many reasons that are unrelated to individual income, for example because of tax reforms or for demographic reasons. Third, I present a stylized example that captures the idea behind fiscal insurance.

2.1 Tax Schedule and Income Dispersion

I estimate the US tax schedule using data from the Panel Study of Income Dynamics (PSID) for survey years 2017 and 2023, constructing individual tax liabilities with NBER's TAXSIM program (Feenberg & Coutts, 1993). Pre-tax income is the sum of wages, pensions, dividends, interest, and self-employment income. Federal tax liabilities are the sum of federal income taxes and payroll taxes, less transfers and social security income. After-tax income is pre-tax income minus tax liabilities.



(a) Tax schedule in the US.



(b) Pre-tax versus after-tax income

Figure 1: The left panel plots households' tax liabilities against their pre-tax income. Each dot represents the average pre-tax income and taxes within each income percentile. The blue dots refer to PSID survey year 2017, the orange dots to PSID survey year 2023. The dashed lines are the linear fit. The right panel plots after-tax income against pre-tax income. The dashed line is the 45-degree line. Units are in $1000. I construct individual tax liabilities using NBER's TAXSIM program. I report federal taxes only, as these back government debt.

Figure 1a plots individual tax liabilities against pre-tax income for both survey years. Each dot represents the mean within each income percentile. First, the tax schedule in the US is affine: the intercept is negative and statistically significant in both cases. A linear fit provides a very good approximation of the relation between pre-tax income and taxes paid ($R^2 = 0.996$ in 2017 and $R^2 = 0.994$ in 2023). Second, the intercept and the slope move over time. The tax schedule in 2023 lies below the schedule in 2017 and is flatter. These patterns are broadly consistent with Heathcote et al. (2017) and Bhandari, Evans, Golosov, and Sargent (2017b): an affine tax technology is a good approximation, but the progressivity varies over time.

Similarly, Figure 1b compares pre-tax and post-tax income, again for both survey years. Taxes compress the cross-sectional distribution of income, and the more so when taxation is more progressive. At low levels of income, households get some income from transfers, whereas at high levels of income households' after-tax income is less than pre-tax income: progressive taxes impact households' exposure to idiosyncratic risk and compress the cross-sectional dispersion.

2.2 Sources of Tax Revenue and Tax Reforms

I next provide anecdotal evidence that the tax structure varies over time in ways that expose different households differently to tax changes. Figure 2 shows that while the share of revenue from each source tends to be persistent, it also features large secular variation. In the first half of the 1900s, excise taxes accounted for half of total revenue, while social insurance played only a minor role. After WWII, the composition shifted dramatically: social insurance and individual income taxes became the dominant sources of revenue. Meanwhile, the share from corporate taxes has steadily declined. These patterns illustrate that the tax structure evolves over time, creating uncertainty about which households will ultimately bear the burden of repaying debt.

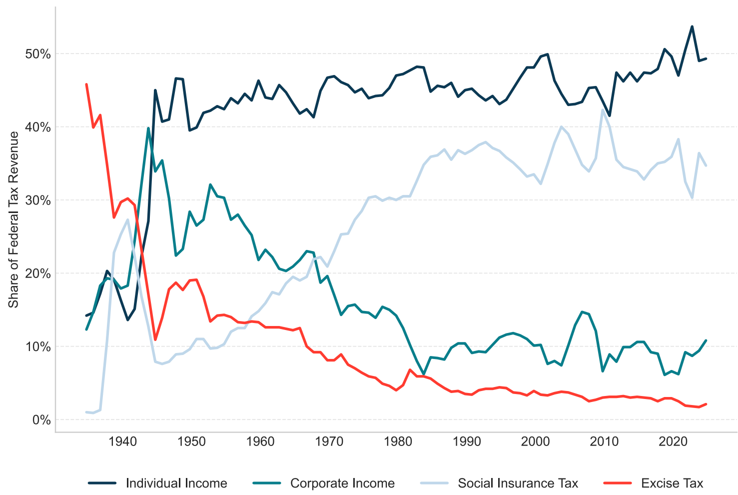


Figure 2: Share of Federal revenue across tax instruments. Data is from The Office of Management and Budget historical Tables.

Figures 10 and 11 in Appendix B.2 show that marginal tax rates on individual and corporate income fluctuate substantially over time, creating uncertainty about both the level of taxes and the degree of progressivity. Moreover, different tax instruments affect households differently depending on their position in the wealth distribution. An increase in payroll taxes places the burden of debt primarily on workers, whereas an increase in capital income or corporate taxes disproportionately affects wealthier households who derive more income from capital (see figures 12a and 12b in the Appendix). The frequency and magnitude of tax reforms reinforce this uncertainty. Appendix B.1 provides a summary of major tax reforms, each involving multiple provisions that shift the relative tax burden across households in heterogeneous ways. While Figure 2 highlights historical patterns, current discussions about fiscal sustainability suggest that future tax adjustments to service rising government debt levels are likely. Large fiscal adjustments in the United States appear particularly plausible given that the US is now a relatively low-tax country compared to other advanced economies (Mankiw, 2025).

Taken together, these patterns highlight two countervailing forces that shape fiscal insurance. On the one hand, progressive taxes provide insurance by compressing the cross-sectional dispersion in after-tax income. On the other hand, uncertainty about the future distribution of the tax burden creates fiscal risk, and this risk scales with the level of debt when future fiscal adjustments are large. My theory formalizes these mechanisms and studies their implications for asset prices and fiscal capacity. This historical experience motivates the stochastic component of tax incidence in my model: households face uncertainty about their relative tax burden that is distinct from their income risk

2.3 A Stylized Example

I describe a stylzed example that clarifies the basic principle behind fiscal insurance. The setup is intentionally streamlined and abstracts from intertemporal savings to emphasize that the combination of government bonds and taxes effectively constitutes an insurance policy.

Consider two agents $i \in \{1, 2\}$ who face uncertainty about their income e. Preferences are represented by a strictly concave utility function $u(c)$. For simplicity, households have utility over terminal consumption only and the discount rate is one. The random income e takes two values, $e_h > e_l > 0$, each occurring with probability $1/2$, and $\bar{e} = \mathbb{E}[e]$. The two agents' income realizations are independent.

At $t = 0$, agents can trade risk-free bonds at the exogenous gross risk-free rate R. In the private market, bonds are in zero net supply. Alternatively, the government can supply B_0 bonds at price $1/R$ at $t = 0$ and finance the repayment $\tau \cdot (e^1 + e^2) = B_0$ at $t = 1$ through income taxes $\tau \cdot e^i$. Each agent's consumption at $t = 1$ equals their after-tax income plus bond proceeds, i.e. $c^i = e^i + B_0/2 - \tau \cdot e^i$. I already use the fact that agents are symmetric and thus choose the same bond holdings $B_0/2$.

Autarky: Suppose $B_0 = 0$. Taxes are also zero. Since the two agents are identical, there is no trading in the bond market. Households thus consume their income $c^i = e^i$, so that $\mathbb{E}[u(c)] = \frac{1}{2}\left(u(e_h) + u(e_l)\right)$.

Fiscal Insurance: Suppose $B_0 > 0$ so that $\tau(e^1 + e^2) = B_0$. Each agent now consumes

$$c^i = e^i(1 - \tau) + B_0/2 = e^i(1 - \tau) + \tau(e^i + e^j)/2 = e^i(1 - \tau/2) + \tau e^j/2$$

If both households receive the same income $e^1 = e^2$, then consumption is the same as in autarky. However, if $e^1 \neq e^2$, then the debt burden falls disproportionately on the lucky agent, while both agents receive the same bond income. As a result, the combination of debt and taxes transfers resources from the lucky to the unlucky agent, some degree of insurance against income risk.

The following sections will integrate this mechanism in a general equilibrium model with heterogeneous agents and study its effects on bond prices. This insurance generates a rich set general equilibrium implications for the pricing of government debt and asset valuations in general.

3 Analytical Results

I describe the idea of fiscal insurance in a two-period endowment economy. The two-period model serves as a laboratory to isolate the forces through which fiscal policy influences households' pricing kernels. I emphasize that households' exposure to uninsurable risks scales with the quantity of outstanding debt. Using this framework, I characterize how government debt affects the risk-free rate and state prices. Further, I derive conditions under which government debt is net wealth. A necessary condition for whether government debt is net wealth is that covariance between households' marginal utility and their relative tax burden is different from zero. In a two-period endowment economy, the net wealth component of government debt does not directly affect asset prices because households do not have access to any storage technologies. However, the sign of the net wealth contribution signals whether further issuance leads to welfare gains.

3.1 Environment

The two-period model builds on Barsky et al. (1986) and introduces two elements. First, I introduce tax intercept shocks. These shocks model uncertainty about the relative tax burden across households that is unrelated to income shocks. Debt issuance exposes households to redistributive risks that may be independent of individual incomes. Second, I incorporate aggregate risk and allow the government to issue state-contingent debt. State-contingent returns on government debt influence how the cross-sectional dispersion in consumption varies across aggregate states.

Households There are two dates, $t = 0, 1$. Uncertainty is resolved in period $t = 1$, and there is no uncertainty at time $t = 0$. The economy is populated by a continuum of ex ante identical households indexed by $i \in \mathcal{I}$. Households rank consumption streams according to

$$U\left(c_0^i, c_1^i(z, y^i)\right) = u\left(c_0^i\right) + \beta \sum_{z=1}^{Z} \sum_{y=1}^{Y} \pi(z, y^i) u\left(c_1^i(z, y^i)\right) \tag{1}$$

The period utility function $u(\cdot)$ is strictly concave and satisfies Inada conditions. The state of the economy, $s = (z, y^i)$ includes aggregate shocks $z = 1, \ldots, Z$ and idiosyncratic shocks y^i. The states $z = 1, \ldots, Z$ describe the aggregate state of the economy. The leading case sets $Z = 2$. Households also face uninsurable shocks $y^i = 1, \ldots, Y$. Each realization of y^i combines two components so that $y^i = (\phi_1^i, \varphi_1^i)$ is two-dimensional. First, households draw a random income share ϕ_1^i of total output. Second, households draw a tax shock φ_1^i, which I describe below in more detail. The joint density of the aggregate and idiosyncratic states $\pi(z, y^i) = \pi(y^i|z)\pi(z)$ is the same for all households. Further, idiosyncratic shocks are independently and identically distributed across households, so a cross-sectional version of the Law of Large Numbers applies.[1] Hence, the ex ante probabilities $\pi(y^i|z)$

[1]See Krueger and Lustig (2010) and references therein for a formal justification of this assumption.

also correspond to the ex post share of agents drawing each realization.

Technology The sole source of wealth is an exogenous aggregate endowment $\boldsymbol{e} = (e_0, e_1(z))$, which I refer to as aggregate output. Realizations of $e_1(z)$ vary only across aggregate states. In period 0, each household receives an equal share of e_0. In period 1, households draw a random share ϕ_1^i of aggregate output. Each household's income in period 1 is thus $\phi_1^i e_1(z)$.

Financial Markets There are two financial securities, both in zero net supply. There is a risk-free asset that pays one unit of the consumption good at time $t = 1$ and a risky asset that pays an exogenous dividend $d_1(z)$ that depends on the aggregate state z. The state-contingent security can be thought of as an aggregate equity claim or as the stochastic return of a long-term bond. Households can freely trade both assets and do not face any portfolio constraint. I introduce market incompleteness by assuming that households cannot insure against idiosyncratic income and tax shocks.

Budget Constraints Each household's budget constraints are

$$c_0^i + b_0^i p_0 + a_0^i q_0 \leqslant e_0 - T_0$$
$$c_1^i(z, \phi_1^i, \tau_1^i) \leqslant \phi_1^i e_1(z) - T_1^i(z, \phi_1^i, \varphi_1^i) + b_0^i + a_0^i d_1(z)$$

where b_0^i and a_0^i denote positions in the risk-free and risky asset, respectively. In period $t = 0$, each household pays the same taxes T_0. In period t=1, tax liabilities vary by households because of idiosyncratic shocks. The term $T_1^i(z, \phi_1^i, \varphi_1^i)$ denotes households i's tax schedule in period $t = 1$, and it can depend on both of tax and income shocks. In period $t = 0$, households choose consumption and portfolios. In period $t = 1$, households receive income and dividends plus principal repayments $b_0^i + a_0^i d_1(z)$ from their financial assets, which they use to pay taxes and consume.

Government The government issues debt and collects taxes to fund an exogenous stream of government purchases $\boldsymbol{g} = (g_0, g_1(z))$. The government budget constraint is

$$g_0 = T_0 + B_0 p_0 + A_0 q_0,$$
$$g_1(z) + B_0 + A_0 d_1(z) = T_1(z),$$

where B_0 and A_0 denote the government borrowing in the risk-free and state-contingent assets, respectively. The government trades the same assets as the households. I take as given the paths of government purchases $\boldsymbol{g} = (g_0, g_1(z))$ and government borrowing, which I refer to as the debt profile $\mathcal{B} = (B_0, A_0)$. Aggregate taxation $T_1(z)$ adjusts endogenously to satisfy the government budget constraint in every period and state. I study the implications of alternative government financing policies, that is debt profile and taxes, for households' pricing kernels in a competitive equilibrium.

Tax Technology Motivated by the tax structure in the United States, the government is endowment with a tax technology that is affine in households' income. Specifically, the government raises revenue through a combination of output and lump-sum taxes. I denote by $\tau_1^e(z)$ the tax rate on output and by $\tau_1(z)$ the total lump-sum tax collected across households. In period 1, aggregate tax revenue is

$$T_1(z) = \tau_1(z) + \tau_1^e(z)e_1(z). \tag{2}$$

Given a debt profile and government purchases, the government budget constraint determines total fiscal needs $T_1(z)$ but leaves the tax instruments $\tau_1(z)$ and $\tau_1^e(z)$ undetermined. For this reason, I parameterize the revenue share of lump-sum taxes by $\kappa \in [0,1]$. As a result

$$\tau_1(z) = \kappa T_1(z) \tag{3}$$

and

$$\tau_1^1(z)e_1(z) = (1-\kappa)T_1(z) \tag{4}$$

Given revenue $T_1(z)$ and $\kappa \in [0,1]$, equation (3) and (4) determine the lump-sum $\tau_1(z)$ and the output tax rate $\tau_1^e(z)$. This specification nests lump-sum and proportional taxes as special cases, where $\kappa = 1$ and $\kappa = 0$, respectively. Importantly, government revenue only varies across aggregate states.

Individual Tax Schedules Given a tax technology, households also face affine tax schedules. However, while aggregate taxation is free of idiosyncratic risks, individual taxes depend on income ϕ_1^i and tax φ_1^i shocks. Specifically, each household faces an affine tax schedule of the form

$$T_1^i(z, \phi_1^i, \varphi_1^i) = \tau_1^e(z)\left[\phi_1^i e_1(z)\right] + \varphi_1^i \tau_1(z) \tag{5}$$

The first term describes the output tax paid by household i. Lucky households with a good income shock ϕ_1^i earn income $\phi_1^i e_1(z)$ and pay more output taxes $\tau_1^e(z)\left[\phi_1^i e_1(z)\right]$. As a result, output taxes provide insurance against idiosyncratic shocks. The second term captures uninsurable shocks to the tax intercept. Households pay a stochastic share φ_1^i of the total lump-sum tax $\tau_1(z)$ that the government collects. These shocks capture changes in fiscal policy or redistribution preferences that affect households heterogeneously (e.g. single vs. married, entrepreneurs vs. workers).[2] The tax structure of Aiyagari and McGrattan (1998) is a special case in which $\varphi_1^i = 1$ for all households. I refer to this structure a *pure lump-sum*, whereas *stochastic lump-sum* tax refers to the case in which the variance of φ_1^i is strictly positive. I restrict realizations of both income and tax shocks such that, for each aggregate state z

$$\int_{\mathcal{I}} T_1^i(z, \phi_1^i, \varphi_1^i)di = e_1(z)\tau_1^e(z)\int_{\mathcal{I}} \phi_1^i di + \tau_1(z)\int_{\mathcal{I}} \varphi_1^i di = T_1(z) \tag{6}$$

These restrictions ensure that the government always remains solvent. Figure 3 provides a visual summary of the tax technology. The parameter $\kappa \in [0,1]$ determines the lump-sum share of aggregate tax revenue. The idiosyncratic shocks ϕ_1^i and φ_1^i determine who bears the debt burden.

[2] Appendix B describes several examples of tax reforms that have heterogeneous impact on households.

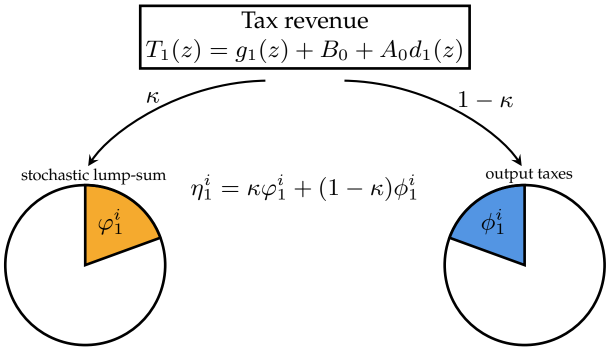


Figure 3: Summary of tax structure. Government purchases and debt determine total tax revenue $T_1(z)$ in period $t = 1$. The revenue share of the stochastic lump-sum tax is $\kappa \in [0, 1]$. Households pay a fraction φ_1^i of the total lump-sum and a fraction ϕ_1^i of the output tax. The relative tax burden is the weighted average between the two uninsurable shocks, where κ determines the weights.

The relative tax burden in period $t = 1$ depends on idiosyncratic shocks. I define η_1^i as household i's relative tax burden, which is the share of total taxes that household i pays

$$\eta_1^i(z, \phi_1^i, \varphi_1^i) \doteq \frac{T_1^i(z, \phi_1^i, \varphi_1^i)}{T_1(z)} = \varphi_1^i \kappa + \phi_1^i (1-\kappa) \tag{7}$$

The relative tax burden is a weighted average of the income shock ϕ_1^i and the tax shock φ_1^i. The tax technology parameter κ determines the weights. Equation (7) shows that output taxation is a special case in which the tax burden and the income shock coincide. The stochastic lump-sum breaks the perfect correlation between income shocks and the tax burden. Higher debt at $t = 0$ affects asset prices because the response of taxes at $t = 1$ determines households' exposure to idiosyncratic shocks.

3.2 Equilibrium

I first examine the case with $Z = 2$, so that markets for aggregate shocks are complete. Section 4.2.3 generalizes the discussion to the case in which markets for aggregate risk are also incomplete. I define a competitive equilibrium with taxes as follows.

Definition 1 (Competitive Equilibrium)**.** *Given the endowment* $\mathbf{e}$*, government purchases* $\mathbf{g}$ *and debt profile* $\mathcal{B} = (B_0, A_0)$*, a competitive equilibrium is a collection of quantities* $(\mathbf{c}^i, a_0^i, b_0^i)$ *and prices* (p_0, q_0) *such that*

1. *The goods market clears in every period;*

$$c_0 \doteq \int_{\mathcal{I}} c_0^i di = e_0 \qquad : \qquad c_1(z) \doteq \int_{\mathcal{I}} \sum_y \pi(y^i|z) c_1^i(z, y^i) di = e_1(z)$$

2. *At time* $t = 0$*, asset markets clear;*

$$\int_{\mathcal{I}} b_0^i di = B_0 \qquad : \qquad \int_{\mathcal{I}} a_0^i di = A_0$$

3. *The aggregate resource constraint* $c_t(z) + g_t(z) = e_t(z)$ *holds in every period;*

4. *Given a tax policy κ, the proportional tax rate τ_t^e and the aggregate lump-sum payment τ_t adjust so that the government budget constraint holds in every period.*

Given that all households face the same strictly concave utility and linear constraints, the equilibrium is straightforward to characterize. Each household chooses identical asset holdings $a_0^i = a_0$ and $b_0^i = b_0$ at time $t = 0$ and consumes an equal share of the aggregate endowment net of government purchases $c_0 = \phi_0(e_0 - g_0)$. Lemma 1 describes individual households' consumption in both periods.

Lemma 1 (Consumption in Two-Period Model)**.** *The equilibrium is symmetric. Each household consumes*

$$c_0^i = e_0 - g_0 \tag{8}$$

$$c_1^i(z, \phi_1^i, \varphi_1^i) = \phi_1^i e_1(z) + B_0\left(1 - \eta_1^i\right) + A_0 d_1(z)\left(1 - \eta_1^i\right) - \eta_1^i g_1(z) \tag{9}$$

Furthermore, each household holds the same portfolio $b_0^i = B_0$ and $a_0^i = A_0$.

In period $t = 0$, all households consume the same share of the aggregate endowment less government spending. Importantly, the equilibrium consumption c_0^i is independent of both the debt profile and taxes. In period $t = 1$, households hold identical portfolios and thus receive identical debt repayments from the government. However, the relative tax burdens η_1^i vary by households because of the idiosyncratic shocks. When tax burdens vary across households ($\eta_1^i \neq 1$), government debt influences households' exposures to uninsurable risks. Moreover, cross-sectional consumption dispersion within each aggregate state scales with the level of debt.

Since c_0^i does not depend on government policy, debt affects asset prices only through its influence on c_1^i. The first term in Equation (9) is household i's endowment income. The second and third terms describe the portfolio return net of taxes paid to finance interest expense. The fourth component reflects the household's share of taxes financing government purchases. Lemma 1 previews how debt and taxes together create insurance. When the tax burden declines with low income shocks ϕ_1^i, debt and taxes together redistribute the burden toward households with high income draws. Without government debt, ex ante identical households cannot achieve risk sharing through private markets. With both assets in zero net supply, markets clear only when $a_0^i = b_0^i = 0$.

3.3 Households' Pricing Kernel

I now describe how government debt and taxes together affect households' pricing kernels and asset prices. I first examine the risk-free rate, then turn to aggregate risk premia. Because markets are incomplete, consumption loads on the idiosyncratic shocks ϕ_1^i and φ_1^i. As usual, the individual pricing kernel corresponds to marginal rate of intertemporal substitution of household i

$$m_1^i(z, \phi_1^i, \varphi_1^i) = \beta \frac{u'(c_1^i(z, \phi_1^i, \varphi_1^i))}{u'(c_0^i)} \tag{10}$$

The individual pricing kernel m_1^i is central to understanding how debt affects asset prices because it captures households' exposure to idiosyncratic risk. In contrast, a representative agent's pricing kernel assigns zero compensation to idiosyncratic risk. Accordingly, individual pricing kernels discount tax liabilities at a different rate than bond income, generating wealth effects.

Importantly, I assume that the government repays debt through tax adjustments. The government adjusts both lump-sum and output taxes such that a fraction $\kappa \in [0,1]$ of total revenue is raised through lump-sum taxes. In practice, governments could also respond by reducing spending or through inflation. I compare how asset prices respond to spending cuts and tax adjustments in Section 3.6.1. Section 4.1.2 introduces nominal debt and explores how distributional concerns change when households worry about debt being inflated away. All proofs are in Appendix A.1.

3.3.1 Precautionary Savings and the Risk-Free Rate

The equilibrium risk-free rate $R_0 = \frac{1}{p_0}$ depends on government debt through a precautionary savings channel.[3] Given a tax share $\kappa \in [0,1]$, debt and taxes together influence households' consumption volatility and thus their demand for risk-free bonds. Proposition 1 shows that the risk-free rate varies with bond supply only if the relative tax burden correlates with the curvature of the period utility function u''. Further, higher government debt raises (lowers) the risk-free rate when it reduces (increases) consumption volatility across idiosyncratic states. Importantly, increases in both risk-free and state-contingent debt affect the risk-free rate. When government debt appreciates in bad aggregate states, risky debt provides better insurance against idiosyncratic shocks by inducing procyclical variation in households' cross-sectional exposure to uninsurable risks.

Proposition 1 (Debt Supply and Asset Prices)**.** *An increase in risk-free borrowing B_0 impacts the price of the risk-free bond p_0 according to*

$$\frac{\partial p_0}{\partial B_0} = -\frac{\beta}{u'(c_0^i)} \mathbb{C}ov\left(u''(c_1^i), \eta_1^i\right)$$

An increase in state-contingent borrowing A_0 impacts the price of the risk-free bond p_0 according to

$$\frac{\partial p_0}{\partial A_0} = \frac{\beta}{u'(c_0^i)} \mathbb{E}\left[u''(c_1^i) d_1^2 \left(1 - \eta_1^i\right)\right]$$

where c_1^i is given in Lemma 1.

Corollary 1 then follows immediately.

Corollary 1 (State-Contingent Debt)**.** *If the tax burden is constant ($\eta_1^i = 1$), p_0 is invariant to debt supply.*

The key message of Proposition 1 is that government debt impacts the risk-free rate when (i) future tax adjustments affect households' exposure to idiosyncratic risk and (ii) households have precautionary demand. As a result, government debt alone does not improve risk sharing. The combination of debt and taxes alters individual pricing kernels. With precautionary savings motives ($u''' > 0$), reduced consumption volatility raises the risk-free rate, while increased volatility lowers it.[4]

[3] A precautionary motive arises when $u''' > 0$; see Kimball (1990). Quadratic utility does not feature precautionary demand, so prices are independent of fiscal policy. However, it also violates Inada conditions.

[4] The conclusion that *risk sharing* depends on the tax structure is robust to ex-ante heterogeneity. Ex-ante heterogeneity also implies that debt has *redistributive* effects because some households hold more government securities.

Market incompleteness is not sufficient to generate supply effects on the risk-free rate. If the tax burden is constant, e.g., with pure (non-stochastic) lump-sum taxes, then $\frac{\partial p_0}{\partial B_0} = 0$ even if households cannot insure against idiosyncratic income risks ϕ_1^i. Households hold government bonds, but they come with equivalent tax liabilities that net out with interest income. Hence, equilibrium consumption equals income and the economy behaves as in a no-trade equilibrium
Second, the risk-free rate varies with supply because fiscal policy interacts with households' precautionary savings.[5] An increase in debt at $t = 0$ triggers a tax adjustment at $t = 1$. The size of the tax adjustment scales with the quantity of government debt outstanding. Higher tax rates lower households' exposure to idiosyncratic risk by shifting the composition of after-tax income toward bond returns and away from risky endowment income.

Issuance of state-contingent debt also impacts the level of taxes, but the effect is more complicated because bond payoffs vary across aggregate states. The magnitude of the fiscal response in period $t = 1$ is now state dependent. Depending on the payoff structure, issuance of state-contingent debt can generate *procyclical* variation in exposure to idiosyncratic risk, providing even better risk sharing at moderate debt levels. Higher taxes in bad times mean that cross-sectional dispersion in income declines precisely when it is needed most. Importantly, this holds even when the cross-sectional variance of idiosyncratic shocks is constant across states.[6]

If state-contingent bonds pay off more in bad aggregate states, taxes rise more in bad times, providing redistribution precisely when it is most valuable. State-contingent debt thus serves as a commitment device to redistribute when it is needed most. This distinguishes state-contingent debt from risk-free debt: while risk-free debt provides constant insurance across all states, state-contingent debt concentrates insurance in states where the marginal value of redistribution is highest. The government would want to structure its debt so that taxes to repay interest expenses increase in bad states. As a result, the government raises taxes precisely when aggregate consumption is already low.

[5] If utility is quadratic, then $u''(\cdot)$ is constant and $\frac{\partial p_0}{\partial B_0} = 0$. Because households only care about the expected consumption, demand for savings is independent of debt supply even though risk sharing improves.

[6] Several papers assume that the variance of idiosyncratic shocks is *countercyclical* to generate variation in risk premia over time. Recent examples are Di Tella (2017), Di Tella (2020), and Brunnermeier et al. (2024). In these papers, the volatility process is exogenous. I show instead that debt supply interacts with uninsurable risk to induce variation in households' equilibrium exposure to idiosyncratic shocks across aggregate states.

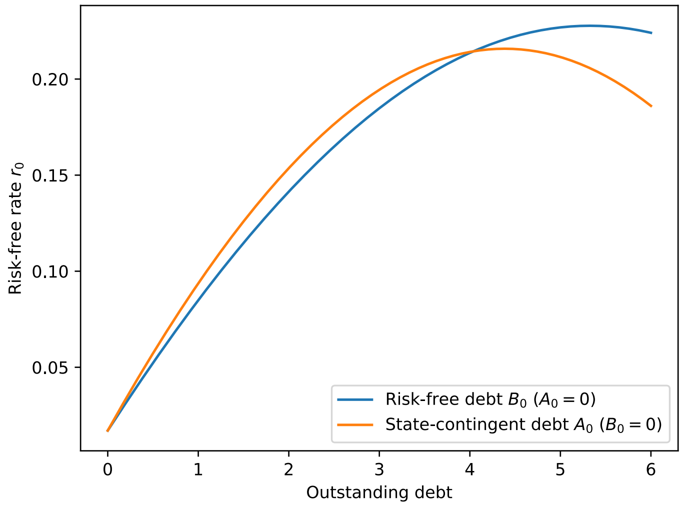


Figure 4: Comparison of risk-free rate $r_0 = \frac{1}{p_0} - 1$ response to an increase in government borrowing. The blue line shows how the risk-free rate varies with the supply of risk-free debt, holding $A_0 = 0$ fixed. The orange line shows how the risk-free rate varies with the supply of risky debt, holding $B_0 = 0$ fixed. To emphasize the insurance effect, I set $\kappa = 0$, i.e. only output taxes, CRRA utility with $\gamma = 2$, and $\beta = 0.96$. I specify the payoff of the state-contingent asset so that it is negatively correlated with output growth.

Figure 4 plots the risk-free rate r_0 against the quantity of outstanding debt. The orange line shows the response of the risk-free rate as B_0 varies while keeping the supply of state-contingent debt $A_0 = 0$ constant. The blue line shows the response of the risk-free rate as A_0 varies while keeping the supply of risk-free bonds $B_0 = 0$ constant. At low levels of debt, the response of the risk-free rate is relatively stronger after an increase in state-contingent debt. Households are better insured against idiosyncratic shocks because there is more redistribution when the aggregate endowment is low. Thus, the risk-free rate increases faster relative to issuance of risk-free debt. One might interpret this as the government supplying more safe assets, but this intuition is misleading. Markets for aggregate risks are already complete, so debt issuance alone cannot affect household consumption.

3.4 The Value of Fiscal Insuarance

I now show that fiscal insurance increases the present discounted value of households' consumption and introduces a valuation gap between households' endowment income and their consumption. This exercise delivers an important lesson by providing conditions under which government debt is positive net wealth in terms of the covariance between marginal utilities and tax burdens. The valuation wedge increases both with the quantity of debt and the shadow price of idiosyncratic risk, so government debt being net wealth is not necessarily desirable: a large net wealth share of government debt means the shadow price of idiosyncratic risk is large and there are untapped gains from insurance. While valuation gaps have no impact in endowment economies, the same logic carries over to richer environments and explains why fiscal insurance lowers investment.

I define a household's initial wealth as the present value of income net of future taxes. In equilibrium, this quantity corresponds to the present discounted value of household consumption. Importantly, present discounted values are computed using households' individual pricing kernels m_1^i. In models with complete markets, all households share the same pricing kernel, making wealth valuation unambiguous. With incomplete markets, households have heterogeneous pricing kernels m_1^i. I use each household's individual pricing kernel to value their wealth: m_1^i reflects their marginal valuation of future income and taxes given their specific (idiosyncratic) risk exposures and consumption profiles.

Definition 2 (Household's Initial Wealth)**.** *A household's initial wealth at time $t = 0$, denoted by W_0^i is*

$$W_0^i = e_0^i + \mathbb{E}_0[m_1^i \phi_1^i e_1^i] - T_0^i - \mathbb{E}_0[m_1^i T_1^i]$$

where the present value is evaluated using the household's individual pricing kernel m_1^i.

Definition 2 emphasizes the dependence of households' individual pricing kernels on the debt profile by writing $m_1^i(\mathcal{B})$. The combination of government bonds and taxes introduces a valuation gap between households' income and their consumption. The gap arises because these two streams have different risk characteristics and because m_1^i compensates exposure to these risks. However, the link between this net value, asset prices, and welfare is more subtle. Proposition 2 shows that the conditional covariance between m_1^i and households' tax burden determines whether fiscal policy influences intertemporal budget constraints.

Proposition 2 (Debt Issuance and Wealth Effects)**.** *Government debt is perceived as net wealth if initial wealth differs from the present value of income net of taxes for government spending, that is, $\Delta(\mathcal{B}) \neq 0$ in*

$$W_0^i = e_0^i + \mathbb{E}_0[m_1^i(\mathcal{B})\phi_1^i e_1^i] - g_0 - \mathbb{E}_0[m_1^i(\mathcal{B})\eta_1^i g_1] + \Delta(\mathcal{B}) = c_0^i + \mathbb{E}_0[m_1^i(\mathcal{B})c_1^i]$$

where

$$\Delta(\mathcal{B}) \doteq B_0 \mathbb{E}_0\left[(1-\eta_1^i)m_1^i(\mathcal{B})\right] + A_0 \mathbb{E}_0\left[(1-\eta_1^i)m_1^i(\mathcal{B})d_1\right]$$

If $B_0 > 0$ and $A_0 = 0$, then $\mathbb{C}ov_0(\eta_1^i, m_1^i) = 0$ implies $\Delta(\mathcal{B}) = 0$. If the government also issues state-contingent debt ($A_0 > 0$), then $\mathbb{C}ov_0(\eta_1^i, m_1^i|z) = 0$ is sufficient for $\Delta(\mathcal{B}) = 0$. If $B_0 > 0$ and $A_0 > 0$, then $\Delta(\mathcal{B}) > 0$ implies that an increase in B_0 lowers the (shadow) price of idiosyncratic risk.

Proposition 2 provides conditions under which the present value of a household's consumption is different from the present value of after-tax income. The valuation gap arises because of households' equilibrium exposure to uninsurable risks: households discount future taxes using their individual pricing kernels. Taxes depend on income and tax shocks (ϕ_1^i and τ_1^i), whereas the return on government bonds only varies across aggregate states, and market prices only reflect compensation for aggregate risks.[7] When the tax burden is low in periods of high marginal utility, tax liabilities are essentially a risky cash flow from the household's perspective. The valuation gap only appears if tax liabilities are valued using individual pricing kernels. Any SDF in the space of tradable payoffs

[7]In representative agent economies, the covariance term is by construction zero because the representative agent pays all taxes. The covariance term is also zero in some heterogeneous agent models provided markets are complete. The additional requirement is that the present value of taxation to each agent is independent of debt issuance policies.

would value the tax liabilities differently. It follows that the market SDF $m_1^* = \mathbb{E}[m_1^i|z]$ prices these assets correctly.[8] The appropriate discount factor for these tax liabilities is m^i, not m^*.

Government debt makes households feel wealthier because the burden of debt repayment falls on lucky households with favorable income shocks. If the relative tax burden declines after a bad shock, taxes become risky claims: households pay more taxes in good (idiosyncratic) states when their marginal utility is low. As a result, households discount taxes at a higher rate than government debt, and consumption exceeds the present value of income. The valuation gap thus reflects exposure to undiversifiable risk: a positive net wealth effect arises precisely because the present value of taxes reflects a positive shadow price of idiosyncratic risk.[9]

While uncertainty about the relative tax burden is necessary for net wealth effects, the converse is not true. Importantly, Proposition 2 does *not* say that uncertainty about the future tax burden implies that government debt is net wealth. A leading example is a situation in which there is uncertainty about future taxes but individual consumption c_1^i varies only across aggregate states.[10] The reason is that households may discount taxes at the same rate as the return on government bonds even if their income is risky, because the shadow price of idiosyncratic risk is zero. Another case is when the relative tax burden and marginal utilities are orthogonal: individual pricing kernels load on η_1^i in a non-linear way, so there exists a debt profile $\mathcal{B}_0$ such that $\mathbb{C}\text{ov}(\eta_1^i, m_1^i(\eta_1^i)|z) = 0$ and $\Delta(\mathcal{B})$ is zero.

To further understand the nature of the valuation gap and whether it washes out in the aggregate, it is useful to compare the present value of aggregate tax revenue under two valuation approaches.[11] The first approach, which is standard in the asset pricing literature, is to discount aggregate taxes directly using the market SDF m_1^*, which gives $T_0 + \mathbb{E}_0[m_1^* T_1]$. The second approach computes the present value of taxes for each household and aggregates such that

$$\int_{\mathcal{I}} T_0^i di + \int_{\mathcal{I}} \mathbb{E}_0[m_1^i T_1^i] di = T_0 + \mathbb{E}_0\left[T_1 \int_{\mathcal{I}} m_1^i \eta_1^i di\right] = T_0 + \mathbb{E}_0[\overline{m}_1 T_1]$$

where I define $\overline{m}_1 \doteq \int_{\mathcal{I}} m_1^i \eta_1^i di$. The term $\overline{m}_1$ can be thought of as a tax-weighted SDF. In complete markets, both approaches yield identical results. If markets are incomplete, then $m_1^* \neq \overline{m}_1$, and the two approaches typically differ. However, $\overline{m}_1$ is not a proper SDF, as it does not price aggregate payoffs correctly. The term $\overline{m}_1$ discounts aggregate cash flows at a rate that is too high because it

[8] The market SDF $m_1^* = \mathbb{E}[m_1^i|z]$ is the unique SDF in the space of tradable payoffs. It is the projection of each individual household's SDF in the span of the government debt portfolio.

[9] The condition $\mathbb{C}\text{ov}_0(\eta_1^i, m_1^i|z) = 0$ is not necessary for $\Delta(\mathcal{B}) = 0$ because no issuance ($A_0 = B_0 = 0$) also implies $\Delta(\mathcal{B}) = 0$. This corresponds to a no-trade (autarkic) equilibrium in which asset markets are inactive (see, e.g., Krueger and Lustig (2010)). This case is uninteresting: the net value of government debt is zero simply because the government issues no debt. Further, I show below that the no-trade equilibrium is inefficient. A benevolent government seeks to achieve $\Delta(\mathcal{B}) = 0$ by setting $\mathbb{C}\text{ov}_0(\eta_1^i, m_1^i|z) = 0$.

[10] This occurs with $\kappa = 0$ if the debt profile $\mathcal{B}$ is chosen such that the implied tax rate on income is 100%, that is, $\tau_1^e = 1$. This is also the case if markets for idiosyncratic shocks are complete.

[11] These ideas share similarities with the *buy-and-hold* perspective and *dynamic trading perspective* proposed by Brunnermeier et al. (2024). In incomplete markets, the present value of an aggregate cash flow need not equal the sum of the present values of its components if they have different exposures to uninsurable risk. While Brunnermeier et al. (2024) apply this concept to the valuation of cash flows from trading bonds, I study the valuation of individual tax liabilities.

reflects the impact of idiosyncratic risk on the risk-free rate. This shadow risk-free rate is higher when taxation insures households against idiosyncratic shocks.

Proposition 2 holds regardless of the particular instruments the government uses to collect taxes: an excise tax on c_1^i would also provide fiscal insurance as long as favorable income shocks lead to higher consumption. Further, wealth effects depend only on the covariance between the relative tax burden and marginal utility, possibly conditional on the aggregate state. A non-zero net wealth share $\Delta(\mathcal{B})$ does not require government debt to be special, e.g., because of liquidity premia (Di Tella, 2020). What matters instead is that government debt affects the equilibrium allocation of risk.

Remark The assumption that agents are ex-ante identical is important because it rules out redistribution through taxation, allowing me to isolate the effects of insurance. Ex-ante symmetry requires households to face the same tax schedule and the same uncertainty about the relative tax burden. Specifically, all households must draw the tax shock from the same distribution φ_1^i. This formulation connects my analysis to existing work exploring how government debt affects risk sharing in closed economies (Brunnermeier et al., 2024; Di Tella, 2020), in which tax changes are entirely unanticipated.

3.5 Connection with Households' Welfare

In the two-period model, government debt impacts asset prices because fiscal policy affects the equilibrium allocation of uninsurable risks, and thus households' pricing kernel. However, because there are no storage technologies, asset prices do not depend directly on $\Delta(\mathcal{B})$. In contrast, $\Delta(\mathcal{B})$ is useful to understand the effect on government debt on risk sharing and welfare. The term $\Delta(\mathcal{B})$ connects to the first-order conditions of a benevolent government that chooses a debt profile $\mathcal{B}$ to maximize households' welfare while taking the affine tax technology (2) as given. Importantly, government debt being perceived as *positive* net wealth is not necessarily desirable, as a positive net wealth share reflects untapped welfare gains from further redistribution of idiosyncratic risk.

The net contribution of government debt to households' wealth can be written as the product of the quantity of insurance and the marginal value of insurance. The quantity of insurance reflects the size of government debt. In contrast, the marginal value of insurance reflects the welfare gain from an incremental increase in government debt. The net wealth effect $\Delta(\mathcal{B})$ is zero if either of these terms are zero, yet the welfare implications differ sharply. If debt supply is zero, there is no trading in financial assets and the equilibrium features autarky. Hence, consumption volatility is maximal. If the marginal value of insurance is zero, consumption volatility attains the minimum a government can achieve under the tax constraint. To formalize this argument, I link the present value contribution of interest expense $\Delta(\mathcal{B})$ to the first-order conditions from a constrained planner problem in which the government chooses a debt profile $\mathcal{B}$ but takes the tax structure $\kappa \in [0, 1]$ as given.[12]

[12]By *constrained*, I mean that the government takes the tax schedule as given. In contrast, the Mirrleesian approach endogenizes the menu of tax instruments and does not restrict the government to set a tax schedule that is linear on income, see Kocherlakota (2005) and Golosov et al. (2003). There are many other ways to implement the same allocation if the government can choose the tax schedule as well. However, that would restore a version of Ricardian equivalence that

I adopt a utilitarian welfare criterion that treats all households symmetrically. Dávila, Hong, Krusell, and Ríos-Rull (2012) present a justification of this criterion in the context of incomplete market models. Accordingly, the objective of the government is

$$\max_{B_0, A_0} \mathcal{W}_0 = \int_{\mathcal{I}} u(c_0^i) + \beta \sum_z \sum_y \pi(z, y^i) u(c_1^i(z, y^i)) di \tag{11}$$

where consumption coincide with the outcome of the competitive equilibrium in Lemma 1

$$\begin{aligned} c_0^i &= e_0 - g_0 \\ c_1^i(z, \phi_1^i, \varphi_1^i) &= \phi_1^i e_1(z) + B_0 \left(1 - \eta_1^i\right) + A_0 d_1(z) \left(1 - \eta_1^i\right) - \eta_1^i g_1(z) \end{aligned}$$

and, further, the government is subject to the affine tax technology

$$\begin{aligned} T_1(z) &= \tau_1(z) + \tau_1^e(z) e_1(z) \\ \tau_1(z) &= \kappa T_1(z) \qquad : \qquad \kappa \in [0, 1] \end{aligned}$$

The government faces a trade-off between providing insurance against income risk and exposing households to fiscal risk: an optimal debt portfolio balances these two forces. Importantly, the government has no control over the revenue share of lump-sum taxes or over the distribution of tax shocks φ_1^i. Proposition 3 characterizes the first-order conditions for the optimal debt profile $\mathcal{B}^*$. The assumption that households are ex ante identical simplifies the analysis considerably.

Proposition 3. *Let welfare $\mathcal{W}_0$ be given by (11). The optimal debt profile $\mathcal{B}^* = \arg\max \mathcal{W}_0$ is the solution of*

$$\mathbb{E}\left[m_1^i(\mathcal{B}^*)(1 - \eta_1^i)\right] = 0 \qquad : \quad B_0^* \tag{12a}$$

$$\mathbb{E}\left[m_1^i(\mathcal{B}^*) d_1 (1 - \eta_1^i)\right] = 0 \qquad : \quad A_0^* \tag{12b}$$

In the equilibrium associated with $\mathcal{B}^*$, the present-value contribution of interest expense is zero $\Delta(\mathcal{B}^*) = 0$. The no-trade equilibrium $\mathcal{B}^{nt} = (0, 0)$ also implies $\Delta(\mathcal{B}^{nt}) = 0$. However, $\mathcal{B}^{nt} = (0, 0)$ does not satisfy (12a) and (12b). If tax shocks are fully unanticipated, $\mathbb{C}ov_0(m_1^i, \eta_1^i | z) = 0$ is sufficient and necessary for optimality.

At the optimum, the government chooses a debt profile such that each household's marginal utility is uncorrelated with the tax burden in period $t = 1$.[13] This condition holds, for example, if marginal utility at $t = 1$ only depends on aggregate shocks, or if the relative tax burden is constant. The main message of Proposition 3 is that the covariance between individual pricing kernels and tax burden offers an interesting economic interpretation of $\Delta(\mathcal{B})$ in terms of welfare gains from an additional unit of government debt. Specifically, the sign of the net wealth share of debt determines whether an increase in government debt improves risk sharing. If the tax burden is constant, the welfare gain is always zero regardless of the distribution of idiosyncratic risks.

makes the debt composition irrelevant (see Bassetto and Kocherlakota (2004); Werning (2007)).

[13] Bhandari et al. (2017a) study optimal debt management in incomplete markets. The mainstream view is that government serves to smooth tax distortions over time. In Angeletos (2002), the maturity structure is chosen to complete the markets.

Another way to interpret this is to view $\Delta(\mathcal{B})$ as households' marginal valuation of fiscal insurance.[14] A positive contribution of interest expense to the present value of household consumption, $\Delta(\mathcal{B}) > 0$, signals that demand for insurance is non-satiated. Conversely, a negative contribution means that there is too much government debt.[15] Under this interpretation, government debt serves as a safe asset from individual households' perspective as long as the relative tax burden associated with interest expense declines after bad idiosyncratic shocks. However, since security payoffs are exogenous, the two-period model is silent about what makes government debt safe for the aggregate economy.

Proposition 3 shows that $\Delta(\mathcal{B}) = 0$ does not necessarily imply that government debt does not improve risk sharing across households. Rather, it means that the additional welfare gain from more debt is zero. A zero wealth contribution $\Delta(\mathcal{B}) = 0$ emerges both in autarky and in the optimum. In contrast, a positive contribution to wealth appears when consumption volatility is lower than in autarky, but higher relative to the minimum the government can achieve. The result that $\mathbb{E}_0[m_1^i(\eta_1^i - \eta_0^i)|z] = 0$ is equivalent to saying that marginal utility is orthogonal to the relative tax burden in the subspace spanned by the government's tradable assets. This follows from the government's desire to minimize cross-sectional dispersion in household consumption, constrained by its inability to eliminate that dispersion entirely. Government debt being perceived as positive net wealth is not necessarily desirable from an households' perspective, as it may reflect scarcity of safe assets. The following two special cases illustrate how a government trading a complete set of securities can achieve $\Delta(\mathcal{B}^*) = 0$. However, only in one case the government can choose a debt portfolio to entirely neutralize uninsurable risk on behalf of the households and satiate demand for safety.

This discussion suggests that government debt can move the economy away from a no-trade equilibrium that would prevail under autarky ($A_0 = B_0 = 0$). However, households benefit from holding government debt only when the tax structure supports a different allocation. Debt alone does not improve risk sharing, but rather becomes valuable through its interaction with taxation.[16]

Figure 5 illustrates the solution to equations (12a) and (12b) under alternative assumptions about the volatility of idiosyncratic shocks and the structure of the tax technology. Circles denote the benchmark case in which the volatility of income and tax shocks coincide. The optimal debt composition reflects a trade-off between providing insurance against idiosyncratic income risk and limiting exposure to fiscal risk. Higher future taxes increase insurance but also amplify fiscal uncertainty, so the government's optimal portfolio balances these opposing effects.

[14] Interpreting the net value $\Delta(\mathcal{B})$ as demand for insurance leads to an analogy between the planner's problem (11) and the Friedman Rule for liquidity demand. A government seeks to satiate demand for insurance so that $\Delta(\mathcal{B}) = 0$ in the same way a central authority should provide liquidity until satiation. In an endowment economy, there is no cost in doing so.

[15] I am restricting my analysis to the empirically relevant case of non-negative issuance $B_0 \geqslant 0$ and $A_0 \geqslant 0$, thereby ignoring situations in which government is a net lender.

[16] Similar ideas appear in Constantinides and Duffie (1996), though they do not consider how government debt can improve upon autarkic allocations.

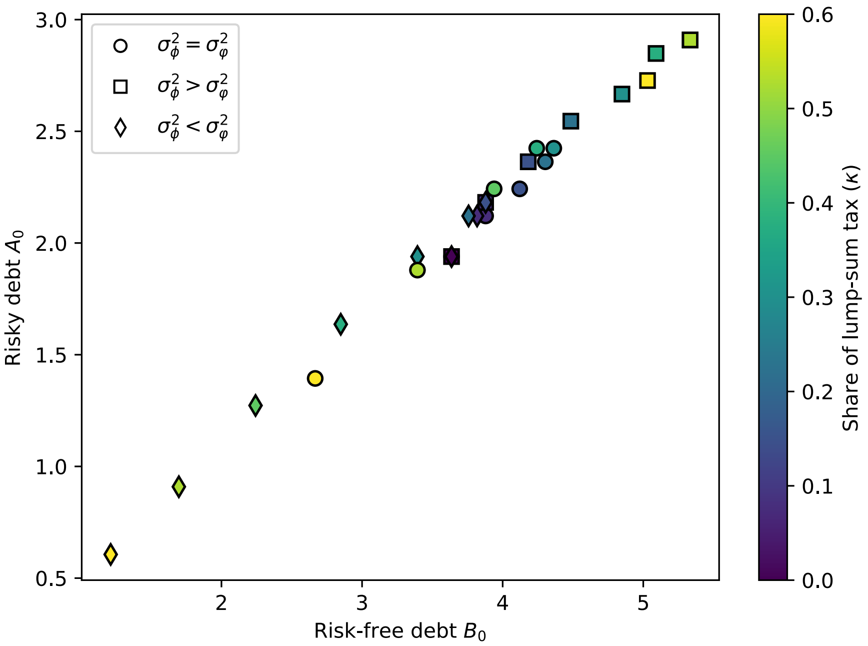


Figure 5: Optimal composition of government debt profile $\mathcal{B}^* = (B_0, A_0)$ for different specifications of the volatility of idiosyncratic shocks and tax technology. The horizontal axis plots the quantity of risk-free debt outstanding, B_0. The vertical axis plots the quantity of state-contingent debt outstanding, A_0. The color scale describes the tax technology. Blue (yellow) markers reflect a low (high) share of stochastic lump-sum taxes. The shape of the market reflects the relative dispersion in income versus tax shocks: circles indicate that the volatility is the same $\sigma^2_\phi = \sigma^2_\varphi$, squares indicate that income shocks have higher volatility $\sigma^2_\phi > \sigma^2_\varphi$, diamonds indicate that tax shocks have higher volatility $\sigma^2_\phi > \sigma^2_\varphi$. I set CRRA utility with $\gamma = 2$, and $\beta = 0.96$. I specify the payoff of the state-contingent asset so that it is positively correlated with output growth.

As the tax technology becomes more reliant on stochastic lump-sum taxation (i.e., as κ increases), the insurance motive weakens relative to the fiscal risk, and the equilibrium level of debt declines. When income shocks are more volatile than tax shocks, the government finds it optimal to issue more debt, both state-contingent and risk-free, to provide greater insurance against idiosyncratic income fluctuations. Conversely, when tax shocks are more volatile, the fiscal-risk channel dominates, and the government reduces its average debt holdings.

Projection Interpretation Since there are no investment distortions, all effects of government debt issuance operate through the pricing kernels. The government's problem can be viewed as choosing a portfolio (B_0, A_0) that minimizes the variance of households' marginal utility. The optimality condition $\mathbb{E}_0[m_1^i(\eta_1^i - 1)|z] = 0$ must hold because any correlation would allow the government to reduce the variance of marginal utility by adjusting its asset portfolio. However, marginal utility is driven by both aggregate shocks z and idiosyncratic shocks y, while the government's debt portfolio spans only aggregate shocks z. Hence, the government cannot eliminate all sources of marginal utility variation.

The condition $\mathbb{E}_0[m_1^i(\eta_1^i - 1)|z] = 0$ ensures that marginal utility is orthogonal to the relative tax burden in the subspace spanned by government bonds. This is equivalent to projecting marginal utility onto the space of tradable government assets and setting the correlation with tax burdens to zero. Since households can hedge against aggregate risk through private markets, the government's role is to provide fiscal insurance against the uninsurable components of households' income. The optimal debt portfolio achieves the best possible risk sharing subject to the constraint that only aggregate-state-contingent instruments are available. Any correlation between marginal utilities and taxes would represent unexploited insurance opportunities within the government's feasible set.

3.5.1 Special Cases

I now discuss two special cases with analytical solutions for the optimal debt structure. The key idea is that a government should choose the debt structure $\mathcal{B}^*$ in a way that trades-off insurance of income shocks ϕ_1^i against exposure to fiscal risks φ_1^i.

Example 1 (Proportional Taxation Only): Suppose $\kappa = 0$, so the government raises tax revenue through output taxes only. The relative tax burden is $\eta_1^i = \phi_1^i$. Hence, the relative tax burden and the idiosyncratic income shock are perfectly correlated. Consumption at time $t = 1$ is

$$\begin{aligned} c_1^i(z, \phi_1^i, \varphi_1^i) &= \eta_1^i e_1(z) - \eta_1^i g_1(z) + (1 - \eta_1^i) B_0 + (1 - \eta_1^i) A_0 d_1(z) \\ &= \eta_1^i \left[e_1(z) - g_1(z) - B_0 - A_0 d_1(z) \right] + B_0 + A_0 d_1(z) \end{aligned}$$

With $\kappa = 0$, there is no stochastic lump-sum tax, so the government seeks to minimize the consumption loading on income shocks across states. This outcome can be achieved by taxing all output, so that $T_1(z) = e_1(z)$ implies a 100% output tax $\tau_1^e(z) = 1$. The optimal debt profile $\mathcal{B}^* = (B_0^*, A_0^*)$ is the solution of $e_1(z) = g_1(z) + B_0 + A_0 d_1(z)$, $z = 1, 2$. The government improves risk sharing on behalf of households by fully taxing income ($\tau_1^e(z) = 1$) and redistributing tax revenue in equal proportions through interest payments. At the welfare optimum, households' net-of-tax income comes entirely from interest on bonds, and consumption varies only across aggregate states. Hence, $\Delta(\mathcal{B}^*) = 0$ and the government can implement an allocation with full idiosyncratic risk sharing.

Example 2 (Stochastic Lump-sum Tax): Suppose $\kappa = 1$, so the government raises tax revenue through stochastic lump-sum taxes only. The relative tax burden $\eta_1^i = \varphi_1^i$ equals the idiosyncratic tax shock and is independent of income shocks ϕ_1^i. Households' equilibrium consumption is

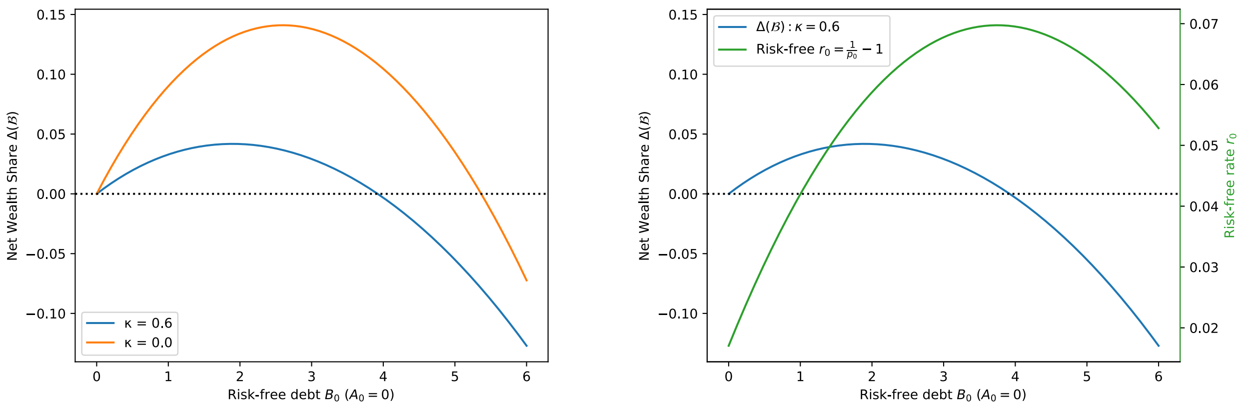
$$c_1^i = \phi_1^i e_1(z) - \eta_1^i g_1(z) + [1 - \eta_1^i] B_0 + [1 - \eta_1^i] A_0 d_1(z)$$

The pricing kernel and the tax burden are orthogonal if $g_1(z) + B_0 + A_0 d_1(z) = 0$ for all $z = 1, 2$; given the government raises no taxes, the tax burden is constant. Households face two types of uninsurable shocks, but the government cannot provide insurance against income shocks. As a result, the government chooses a debt portfolio that minimizes risk exposure under its control, i.e., φ_1^i, so that $T_1(z) = 0$ in all states. As a result, $\Delta(\mathcal{B}^*) = 0$ but the volatility of consumption is the same as in autarky. This is the lowest consumption dispersion the government can achieve.[17]

[17] The effect of a stochastic lump-sum is immediate when $g_1(z) = 0$. The cross-sectional variance of idiosyncratic shocks is $\mathbb{V}\text{ar}_0(T_1^i) = \mathbb{V}\text{ar}_0(\eta_1^i [B_0 + A_0 d_1])$. Conditional on z, the variance is $(B_0 + A_0 d_1)^2 \mathbb{V}\text{ar}(\eta_1^i | z)$. Households' exposure to idiosyncratic shocks scales with the level of debt.

3.6 Numerical Example

I illustrate the relation between the net wealth contribution of government debt $\Delta(\mathcal{B})$ and asset prices. There are two aggregate states $z \in (z_l, z_h)$. The tax and income shocks both take two values such that $\phi_1^i \in (0.75, 1.25)$ and $\varphi_1^i \in (0.75, 1.25)$. I assume $\pi(z, \phi_1^i, \varphi_1^i) = 0.125$ for all states. The endowment is $e_0 = 5$, $e_1(z_l) = 5$ and $e_1(z_h) = 6$. Government purchases are set to zero $\boldsymbol{g} = 0$. The risky asset pays $d_1(z_l) = 0.7$ and $d_1(z_h) = 1.2$. Households have CRRA utility $u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$, with $\beta = 0.96$ and $\gamma = 2$. I compare two settings. First, the government only collects output taxes ($\kappa = 0$), so that $\eta_1^i = \phi_1^i$. Second, the government collects some revenue through (stochastic) lump-sum taxes ($\kappa = 0.6$).



(a) Comparison of $\kappa = 0$ and $\kappa = 0.6$.

(b) Relation between $\Delta(\mathcal{B})$ and r_0.

Figure 6: The left panel plots the net wealth share of government debt $\Delta(\mathcal{B})$ as a function of the supply of risk-free debt for alternative tax technologies. The orange line illustrates the case of output taxes only ($\kappa = 0$), whereas the blue line illustrates the case of both stochastic lump-sum and output taxes ($\kappa = 0.6$). The right panel plots the risk-free interest rate and the net wealth share of government debt for the case $\kappa = 0.6$. The blue line is the same in both figures. I set CRRA utility with $\gamma = 2$, and $\beta = 0.96$. I specify the payoff of the state-contingent asset so that it is positively correlated with output growth.

Figure 6a compares the net wealth contribution of government debt for different tax structures. The orange line plots $\Delta(\mathcal{B})$ against B_0 for the case in which the government only raises output taxes ($\kappa = 0$). At low levels of debt, $\Delta(\mathcal{B})$ is positive and increases with B_0. At very high levels of debt, however, the contribution turns negative. At that point, the government has fully exhausted its role as insurance provider. Further, the point at which the contribution becomes negative is lower if debt issuance exposes households to fiscal risks. The blue line plots $\Delta(\mathcal{B})$ against B_0 under the assumption that the government raises both output and risky lump-sum taxes ($\kappa = 60\%$). Uncertainty about who will bear the debt burden that is unrelated to income shocks erodes the value of fiscal insurance and implies a lower debt threshold above which further debt issuance hurts households.

The main message of Proposition 2 is that the sign of $\Delta(\mathcal{B})$ signals whether further debt issuance improves risk sharing, and thus raises welfare. If $u'''(\cdot) \neq 0$, an improvement in risk sharing translates into a higher risk-free rate because of a reduction in precautionary demand. This connection is shown in Figure 6b. The blue line is the same as in the left panel. The green line plots the risk-free rate r_0 against debt supply. The risk-free rate increases as long as $\Delta(\mathcal{B})$ is positive, but starts to decline when

$\Delta(\mathcal{B})$ turns negative.

3.6.1 Tax Adjustment versus Spending Cuts

Conforming to a long tradition in macroeconomics that takes the path of government spending as given, my analysis explores equilibrium asset prices by assuming the government repays its debt by raising taxes in period $t = 1$. While survey evidence suggests households do perceive fiscal adjustments to be important (Bianchi et al., 2025), raising taxes is not the only way governments may respond to higher debt levels. Governments might instead cut spending or inflate away nominal debt. An important question is whether asset prices respond differently to debt increases depending on how the government chooses to repay its obligations. This section compares the asset pricing response when the government cuts spending while keeping future taxes constant. This setting corresponds either to the government using spending to finance a war abroad, or to a special case of preferences in which the marginal utility of consumption is independent of government spending.

I study a policy in which the government sets the output tax rate to $\tau_1^e = 25\%$ and the lump-sum tax to $\tau_1 = 1$, then cuts government spending to balance the budget. Households' consumption is

$$c_0^i = e_0 - g_0$$
$$c_1^i = \phi_1^i e_1(z)(1 - \tau_1^e(z)) + B_0 + A_0 d_1(z) - \varphi_1^i \tau_1(z)$$

While tax adjustments interact with precautionary demand, spending cuts holding taxes fixed do not. The spending cut impacts the risk-free rate through consumption growth but not through precautionary demand: lower government spending acts as a positive aggregate endowment shock that raises households' consumption. The relation between the risk-free rate and the supply of risk-free debt is increasing and convex as long as $u'' > 0$. The response of asset prices is thus very different compared to a situation in which future taxes increase, as there is no debt level beyond which the risk-free rate starts to fall again.

Different fiscal adjustment mechanisms, i.e. tax increases versus spending cuts, generate *qualitatively* different asset price responses to an increase in government debt supply. As a result, yield responses around QE announcements (Krishnamurthy & Vissing-Jorgensen, 2011), Congressional budget resolutions (Weigand, 2025), or Treasury auctions (Ray, Droste, & Gorodnichenko, 2024) reveal what households expect: future growth or future tax increases. The role of government debt in improving risk sharing depends critically on households' expectations about future policy responses.

4 Extensions and Robustness

Motivated by the current institutional framework in the U.S. Treasury market, I extend the analysis in three main directions. First, I incorporate inelastic foreign investors that do not pay taxes. Second, I introduce nominal debt and briefly discuss if inflationary shocks may lead to different allocation of risk. Third, I study an infinite horizon economy in which the covariance between tax burdens and individual pricing kernels still determines whether government debt is net wealth. I explore the link between uncertainty over taxes and violations of the transversality condition on the government portfolio to understand the relation between fiscal insurance and rational bubbles. The infinite horizon discrete-time analysis serves as a bridge to the dynamic environment in Section 5.

4.1 Extensions

I extend the model to study foreign investors, countercyclical income risk, and rule-of-thumb households. Foreign investors bring in heterogeneity in exposure to fiscal risks, as they pay no taxes.

4.1.1 Foreign Investors

Foreign investors differ from domestic investors in two respects. First, they do not pay taxes. Second, I assume they have downward-sloping demand curves for risk-free bonds.[1] I model their demand as

$$F(p_0) = \alpha_F - \beta_F p_0$$

Bond demand is an affine function of the price p_0. The intercept α_F denotes the average level of foreign demand and β_F is the slope. The market clearing condition for the risk-free bond now becomes

$$\int_{\mathcal{I}} b_0^i di + F(p_0) = B_0$$

As a result, because domestic investors are ex ante identical, $b_0^i = \phi_0^i (B_0 - F(p_0))$. However, the entire tax burden still falls on domestic households. Market clearing in the state-contingent asset is the same as above. Proposition 4 describes equilibrium consumption with foreign investors. By assuming that only domestic investors are marginal, and that foreign investors only trade risk-free debt, the structure of equilibrium remains relatively simple.

Proposition 4 (Equilibrium with Foreign Investors)**.** *In the economy with foreign investors, the consumption of domestic households is given by*

$$\begin{aligned} c_0^i &= e_0 - g_0 \\ c_1^i &= \phi_1^i e_1(z) + A_0 d_1(z)\left(1 - \eta_1^i\right) - \eta_1^i g_1(z) - F(p_0) - (1 - \eta_1^i) B_0 \end{aligned}$$

where $F(p_0) = \alpha_F - \beta_F p_0$. The equilibrium price of the risk-free bond p_0 solves the fixed point

$$p_0 = \frac{\beta}{u'(c_0^i)} \mathbb{E}\left[u'(c_1^i(p_0))\right]$$

If foreign investors are inelastic ($\beta_F = 0$), period $t = 1$ consumption declines in the level of foreign demand α_F.

The relevant pricing kernel for government debt is given by domestic households' marginal utilities. However, foreign investors erode the value of fiscal insurance. Specifically, the government raises

[1] A large share of foreign demand comes from private and public institutional investors. Foreign government investors hold large position in short duration assets, which I interpret here as the risk-free asset (Tabova & Warnock, 2024).

taxes from domestic investors to finance interest payments to foreign investors. As a result, households pay more taxes relative to their interest income, and the value of insurance declines. The point is not just that foreigners hold debt, but that they hold assets without contributing into the tax system.

Figure 7 shows how the parameters of foreign demand impact the welfare effects of fiscal insurance. Households' welfare is defined as in equation (11) and the debt profile is held fixed at $\mathcal{B} = (B_0 = 2, A_0 = 2)$. For a given slope of foreign demand, welfare declines with the average level of foreign demand (α_F). The intuition is clearest in the case where foreign investors are fully inelastic ($\beta_F = 0$). An increase in foreign demand means domestic households hold fewer bonds in their portfolios, so their interest income in period $t = 1$ declines. Foreign investors do not pay taxes, so the entire debt burden still falls on domestic households. As a result, an increase in government debt generates the same fiscal adjustment and thus the same consumption dispersion across shocks. However, lower interest income reduces the average level of consumption. Specifically, issuance of risk-free debt lowers consumption equally in all states. Government debt thus still provides insurance but at the cost of transferring resources abroad: the insurance benefit comes at the expense of lower average consumption in period $t = 1$.

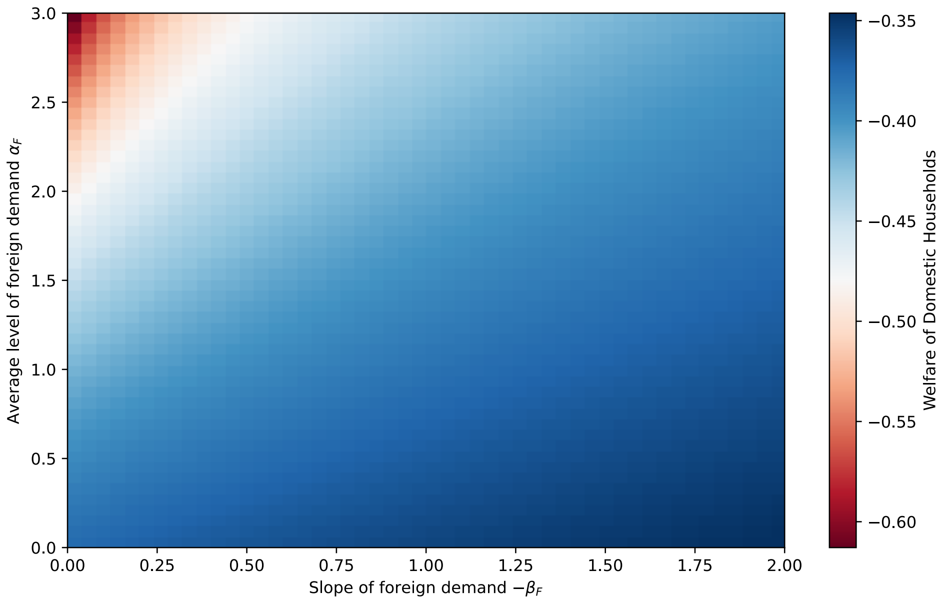


Figure 7: The figure illustrates how welfare depends on the parameters of foreign demand. The horizontal axis plots the negative of the slope coefficient β_F. The vertical axis plots the intercept of foreign demand α_F. The color scale illustrates households' welfare given by the utilitarian criterion (11). I set the debt profile such that $A_0 = 2$ and $B_0 = 2$. Red (blue) indicates low (high) welfare. To emphasize the insurance effect, I set $\kappa = 0$, i.e. only output taxes, CRRA utility with $\gamma = 2$, and $\beta = 0.96$. I specify the payoff of the state-contingent asset so that it is positively correlated with output growth.

On the other hand, the welfare loss from a large foreign sector is lower when foreign demand is very elastic. The expected decline in future consumption incentivizes domestic households to save more in period $t = 0$, lowering interest rates and raising bond prices. An elastic foreign sector reacts to higher prices by reducing demand, so domestic households end up holding more government debt in equilibrium. This effect mitigates the welfare loss from a large foreign sector.

Figure 13 in Appendix C.1.1 compares the impact of foreign investors on the risk-free rate across two tax regimes. The left panel (Figure 13a) illustrates an economy in which the government only collects output taxes. The blue line plots the risk-free rate against the supply of risk-free bonds B_0 in the benchmark without foreign investors. The orange line shows r_0 when foreign investor demand is price inelastic ($\alpha_F = 0.5$, $\beta_F = 0$). The risk-free rate is lower because foreign demand crowds in domestic bond demand as expected consumption growth declines and domestic households seek to save more. The green line shows r_0 when foreign investors are also price sensitive ($\alpha_F = 0.5$, $\beta_F = 2$). Two forces now operate. The level of foreign demand makes fiscal insurance more expensive for domestic investors and lowers interest rates. This pushes away foreign investors and attenuates the adverse impact of foreign demand on welfare. The right panel (Figure 13b) considers a combination of lump-sum and output taxes. At moderate debt levels, foreign demand again lowers the risk-free rate.

The general message is that foreign demand complements domestic bond demand, but only when it is very inelastic. When foreign demand is also very elastic, the initial increase in domestic savings crowds out foreign investors. Because the fiscal adjustment is the same even with foreign investors, government debt still provides insurance. Yet, higher debt also leads to lower consumption growth, so whether the risk-free rate increases depends on which effect dominates. Importantly, the welfare gain from higher insurance must be weighed against the welfare loss from lower future consumption, so the debt threshold beyond which government debt no longer improves welfare will be lower.

4.1.2 Nominal Debt and Distributional Consequences of Inflation

My analysis so far assumes that the government issues real bonds and that higher debt leads to future tax adjustments. However, governments typically issue nominal bonds, which are subject to inflation risk, introducing an additional source of uncertainty for households. The distributional consequences of inflation can differ substantially from those of future fiscal adjustments, as inflation affects real wealth differently depending on households' nominal holdings.

Moreover, the impact on aggregate risk premia depends on whether inflation occurs in good or bad states of the world, potentially amplifying or dampening precautionary demand. Although the government does not issue any risk-free real assets, households can still trade risk-free real debt among themselves, allowing them to agree on a shadow risk-free rate that guides pricing and risk-sharing in the economy. Incorporating nominal bonds and inflation risk would therefore extend the framework to capture how the interplay of debt composition, state-dependent inflation, and household heterogeneity shapes both risk premia and the distributional effects of fiscal policy.

4.1.3 Rule-of-Thumb Households

Fiscal insurance requires households to understand the government budget constraint. Yet, asset demand might display inertia or be subject to other frictions. I briefly explore what happens to risk sharing when some households are non-Ricardian. Suppose a measure μ of rule-of-thumb households do not optimally choose portfolio holdings. These households' budget constraint in period 0 is

$$\bar{c}_0^i + \bar{a}_0^i q_0 + \bar{b}_0^i p_0 = \phi_0^i e_0 - T_0^i$$

where the asset positions $\bar{a}_0^i$ and $\bar{b}_0^i$ are given parameters.[2] The remaining $1 - \mu$ households behave as in the baseline model. The market clearing condition for consumption goods is

$$\mu \bar{c}_0^i + (1 - \mu) c_0^i = e_0 - g_0$$

Market clearing in the asset markets is

$$\mu \bar{b}_0^i + (1 - \mu) b_0^i = B_0$$
$$\mu \bar{a}_0^i + (1 - \mu) a_0^i = A_0$$

Proposition 5 characterizes the pricing kernel of the unconstrained households and describes how a shift in debt supply impacts asset prices.

Proposition 5 (Rule-of-Thumb Investors). *In the economy with rule-of-thumb households, the consumption of unconstrained households is given by*

$$c_0^i = \frac{1}{1-\mu}\left(e_0 - g_0 - \mu \bar{c}_0^i\right)$$
$$c_1^i(z, y^i) = \phi_1^i(z, y^i) e_1(z) + a_0^i d_1(z) + b_0^i - \eta_1^i(z, y^i)[g_1(z) + A_0 d_1(z) + B_0]$$

where the asset positions of the unconstrained households are

$$b_0^i = \frac{1}{1-\mu}\left(B_0 - \mu \bar{b}_0^i\right)$$
$$a_0^i = \frac{1}{1-\mu}\left(A_0 - \mu \bar{a}_0^i\right)$$

The main difference between rule-of-thumb households and foreign investors is that rule-of-thumb households consume part of the endowment. Because they are not optimizing, however, it is the consumption of unconstrained households that determines the relevant pricing kernel for pricing government debt. With foreign investors, all the households who are consuming are also marginal.

4.2 Comparison with other Channels

This section compares the mechanism of fiscal insurance to common mechanisms and frictions often studied in the public finance literature. Specifically, I consider borrowing constraints, tax distortions,

[2]Hand-to-mouth households are a special case with $\bar{a}_0^i = \bar{b}_0^i = 0$.

and incomplete markets for aggregate risks.

4.2.1 The Role of Borrowing Constraints

A key assumption behind Proposition 2 is that households do not face portfolio restrictions. In particular, they can freely borrow and lend at the risk-free rate. However, the role of government debt is very different when households cannot borrow at all (e.g., Aiyagari and McGrattan (1998)). With borrowing constraints, an additional Lagrange multiplier appears in the Euler equations when constraints bind. The mechanism in Proposition 2 is still present, but borrowing constraints introduce an additional effect. Changes in the timing of taxes affect which borrowing constraints bind. Therefore, the conditional covariance between taxes and marginal utilities is no longer the unique force that determines whether government debt is net wealth. Borrowing constraints also lead to an equilibrium where there is no trading in financial assets. However, asset valuations are very different if households cannot issue debt or choose not to do so.

Further, unlike money and capital, government debt must always be backed by taxes. If households pay taxes in exact proportion to their interest income, then government debt will not help even when there are binding borrowing constraints. As a result, the extent to which government debt improves welfare is still related to how the debt burden is distributed across households. In a simple two-period model with two agents and borrowing constraints, the tax structure determines the extent to which government debt helps with consumption smoothing when there are borrowing constraints. Government debt helps because the associated tax cut today relaxes borrowing constraints. While debt issuance clearly helps under most realistic tax arrangements, including lump-sum taxes, it is important to emphasize that government debt is valuable precisely because part of the interest proceeds are backed by somebody else's future resources.

4.2.2 Tax Distortions and Deadweight Losses

A long literature explores the role of government debt in smoothing tax distortions over time and across states (Lucas & Stokey, 1983). While my two-period model assumes taxes are purely redistributive and do not affect aggregate output, I argue that tax distortions alone cannot generate wealth effects and improvements in risk sharing in the aggregate. Therefore, the presence of deadweight losses from taxation does not alter the main message of Propositions 2 and 3. To illustrate this point, suppose income tax collection generates a deadweight loss $\theta(\tau_t^e)$ that depends on the income tax rate. The households' intertemporal budget constraint is now

$$c_0^i + \mathbb{E}_0\left[m_1^i c_1^i\right] = \phi_0^i(e_0 - \theta(\tau_0^e)) + \mathbb{E}_0\left[m_1^i \phi_1^i(e_1 - \theta(\tau_1^e))\right] + \Delta(\mathcal{B})$$

where the government budget constraint implies $T_0 = -p_0 B_0 - q_0 A_0$ and $T_1 = B_0 + d_1 A_0$, and for simplicity, I set $\boldsymbol{g} = 0$. The present value of interest expense $\Delta(\mathcal{B})$ still enters as a separate term. Tax distortions that affect aggregate output are capitalized into the present discounted value of income. Importantly, deadweight losses are discounted at the same rate as future risky income. In contrast, future tax liabilities are discounted at a rate that reflects the covariance between tax burdens and marginal utilities. Tax distortions impact output realizations. They reduce the tax base, affect economic growth, and can create additional consumption volatility. However, these effects alone are not sufficient to make government debt net wealth. The crucial element that makes government debt a store of value is that the debt burden falls disproportionately on lucky households.

4.2.3 Incomplete Markets for Aggregate Risks

A long tradition in public finance explores the role of government debt when markets for aggregate risks are incomplete (e.g., Aiyagari et al. (2002); Bhandari et al. (2017a)). This section shows that the source of market incompleteness, whether aggregate or idiosyncratic, has fundamentally different implications for the effect of government debt on welfare.

When markets for aggregate risks are incomplete the government cannot hedge its budget against aggregate shocks. Yet, missing markets for aggregate risks do not necessarily generate wealth effects if they do not affect the relative tax burden in period $t = 1$ and if households all choose the same equilibrium exposure to aggregate risks. Corollary 2 illustrates this distinction by analyzing a special case of the static model with no idiosyncratic risk and only risk-free debt.

Corollary 2 (Incomplete Markets for Aggregate Shocks)**.** *Suppose that $A_0 = 0$, households can only trade risk-free debt, and $\phi_1^i e_1(z) = e_1(z)$ and $\varphi_t^i = 1$. Then $\Delta(\mathcal{B}) = 0$; the net wealth share is zero.*

Because all households are ex ante identical and choose the same portfolio, the loading of period $t = 1$ consumption on the aggregate shocks is uniform for everyone and does not depend on the quantity of government debt outstanding. As a result, Ricardian equivalence holds.

4.3 Infinite Horizon

Before turning to continuous time, I show that the mechanism described above also matters when households can dynamically trade assets. It turns out that the infinite horizon formulation brings clarity about the relation between an household's transversality condition, the transversality condition on government debt, and the present value of interest expense. Further, a dynamic model allows me to study how uncertainty about the future path of government debt supply impacts asset prices.

4.3.1 Extended Framework

I extend the two-period model of Section 3.1 to infinite horizon. Aggregate uncertainty is modelled as a two-state Markov chain $z_t \in \{z_h, z_l\}$ where $z_h > z_l$. Households and government trade a risk-free debt and a short-lived asset with exogenous state-contingent payoff $d(z)$ that only depends on aggregate shocks. The markets for aggregate shocks are thus dynamically complete. Households' dynamic budget constraints are

$$c_t^i + p_t^i b_t^i + q_t^i a_t^i = b_{t-1}^i + d_t a_{t-1}^i + \phi_t^i e_t - T_t^i$$

where p_t and q_t are the prices of the risk-free bond and the state-contingent asset, respectively. As in the two-period model ϕ_t^i is the idiosyncratic income shock of household i. Government debt issuance is described by a sequence of quantities for risk-free and state-contingent debt $\boldsymbol{\mathcal{B}} = (A_t, B_t)_{t=0}^{\infty}$. The path of debt issuance $\boldsymbol{\mathcal{B}}$ can be stochastic, so that there is uncertainty about future fiscal policy. The government budget constraint is

$$g_t + B_{t-1} + A_{t-1}d_t = p_t B_t + q_t A_t + T_t$$

The government issues debt and collect taxes to finance government purchases $\boldsymbol{g} = (g_t)_{t=0}^{\infty}$. Lemma 2 the contribution of interest expenses to the present value of households' i consumption, which gives the infinite-horizon counterpart of $\Delta(\mathcal{B})$ in Proposition 2.

Lemma 2. *Let m_{t+1}^i denote household's i intertemporal marginal rate of substitution and define $m_{t|t+j}^i$ such that $m_{t|t+h}^i = \prod_{j=0}^{h} m_{t+j}^i$ with $m_t^i = 1$. Then, household i's dynamic budget constraint implies*

$$\begin{aligned}\sum_{j=0}^{T} \mathbb{E}_t[m_{t|t+j}^i c_{t+j}^i] + \mathbb{E}_t[m_{t|t+T}^i p_{t+T}(b_{t+T}^i + a_{t+T}^i)] = b_{t-1}^i + a_{t-1}^i d_t + \sum_{j=0}^{T} \mathbb{E}_t[m_{t|t+j}^i \phi_{t+j}^i e_{t+j}] \\ - \sum_{j=0}^{T} \mathbb{E}_t[m_{t|t+j}^i \eta_{t+j}^i g_{t+j}] + \Delta^i(\boldsymbol{\mathcal{B}})_t^T\end{aligned} \tag{13}$$

where the wealth contribution of government interest expenses to

$$\begin{aligned}\Delta^i(\boldsymbol{\mathcal{B}})_t^T = \mathbb{E}_t\left[m_{t|t+T}^i \eta_{t+T}^i \left(p_{t+T}B_{t+T} + q_{t+T}A_{t+T}\right)\right] - \eta_t^i B_{t-1} - \eta_t^i A_{t-1} d_t \\ + \sum_{j=0}^{T} \mathbb{E}_t\left[B_{t+j}\left(p_{t+j} m_{t|t+j}^i \eta_{t+j}^i - m_{t|t+j+1}^i \eta_{t+j+1}^i\right)\right] \\ + \sum_{j=0}^{T} \mathbb{E}_t\left[A_{t+j}\left(q_{t+j} m_{t|t+j}^i \eta_{t+j}^i - m_{t|t+j+1}^i \eta_{t+j+1}^i d_{t+j+1}\right)\right]\end{aligned}$$

In this framework, uncertainty about the future path of government debt $\boldsymbol{\mathcal{B}}$ also impacts the present value of interest expense. As a result, shocks to issuance of government debt impact the pricing kernel. If debt issuance increases in periods of high marginal utility, then B_{t+j} and m_{t+j}^i are positively correlated and the present value contribution of interest expense is discounted at a lower rate.

These effects can be quantitatively important because households' perceived wealth depends on the entire path of future interest expense, not just on uncertainty about next period's taxation. To gain intuition, I consider how uncertainty about future supply of risk-free debt B_{t+j} shapes the present value of interest expense at time $t + j$. To this purpose, the period $t + j$ contribution to $\Delta(\boldsymbol{\mathcal{B}})$ is

$$\Delta_{t|t+j}^i(B_{t+j}) = \mathbb{E}_t\left[B_{t+j} m_{t|t+j}^i \left(\mathbb{E}_{t+j}\left[m_{t+j+1}^i \eta_{t+j+1}^i\right] - p_{t+j}\eta_{t+j}^i\right)\right] \tag{14}$$

The term $\mathbb{E}_{t+j}\left[m_{t+j+1}^i \eta_{t+j+1}^i\right] - p_{t+j}\eta_{t+j}^i$ comes from uncertainty in the relative tax burden at time $t + j + 1$, and it reflects the fiscal insurance mechanism that appears in the two-period model. This term is non-zero as long as the time $t+j$ conditional covariance between the relative tax burden and marginal utility are correlated. Uncertainty about future supply can amplify this effect if debt supply increases in periods of high marginal utility. Debt issuance provides further insurance when it is needed the most, provided that the conditional correlation between the tax burden and marginal utility is negative. It is important to reiterate that wealth insurance channel from debt issuance are present only to the extent that there is uncertainty about the relative tax burden.

4.3.2 Individual Transversality Conditions and Interest Expense

An important question is the behavior of $\Delta^i(\boldsymbol{\mathcal{B}})_t^\infty = \lim_{T\to\infty} \Delta^i(\boldsymbol{\mathcal{B}})_t^T$ and how it relates to net wealth. Jiang et al. (2024) value government debt in terms of no-arbitrage restrictions between future surpluses and the current market value of government debt. Brunnermeier et al. (2024) show that in incomplete markets, individual transversality conditions do not imply a transversality condition for the government debt portfolio. The same insight applies here. Uninsurable income risks imply that households discount their own wealth at a higher rate relative to future surpluses. The main point of this section is that violations of transversality conditions and fiscal insurance both generate net wealth effects, but neither requires or implies the other. This is important in the dynamic economy with capital as the net wealth effect impacts capital accumulation and economic growth.

Proposition 6 shows that household's transversality condition also does not necessarily imply that the present value of interest expense is finite. This occurs because each household's tax claim is not tradable. I then derive conditions under which (i) $\Delta^i(\boldsymbol{\mathcal{B}})_t^T = 0$ and government debt drops from the aggregate present value budget constraint, and (ii) the present value of interest spending is finite and positive, i.e $\lim_{T\to\infty} \Delta^i(\boldsymbol{\mathcal{B}})_t^T < \infty$.

Proposition 6. *Let the present value contribution of interest expense be*

$$\begin{aligned}\Delta^i(\boldsymbol{\mathcal{B}})_t^T = \mathbb{E}_t\left[m^i_{t|t+T}\eta^i_{t+T}\left(p_{t+T}B_{t+T}+q_{t+T}A_{t+T}\right)\right] - \eta^i_t B_{t-1} - \eta^i_t A_{t-1}d_t \\ + \sum_{j=0}^{T}\mathbb{E}_t\left[B_{t+j}\left(p_{t+j}m^i_{t|t+j}\eta^i_{t+j} - m^i_{t|t+j+1}\eta^i_{t+j+1}\right)\right] \\ + \sum_{j=0}^{T}\mathbb{E}_t\left[A_{t+j}\left(q_{t+j}m^i_{t|t+j}\eta^i_{t+j} - m^i_{t|t+j+1}\eta^i_{t+j+1}d_{t+j+1}\right)\right]\end{aligned}$$

Consider the individual transversality condition

$$\lim_{T\to\infty}\mathbb{E}_t\left[m^i_{t|t+T}\left(b^i_{t+T}p_{t+T}+a^i_{t+T}q_{t+T}\right)\right]=0 \tag{iTVC}$$

and the government transversality condition

$$\lim_{T\to\infty}\mathbb{E}_t\left[m^i_{t|t+T}(p_{t+T}B_{t+T}+q_{t+T}A_{t+T}\right]=0 \tag{gTVC}$$

(a) If taxes are constant or tax shocks are unanticipated, then $\mathbb{E}_t[\eta^i_{t+1}] = \eta^i_t$ and $\mathbb{C}\mathrm{ov}_t(m^i_{t+1}, \eta^i_{t+1}|z_{t+1}) = 0$, then (gTVC) implies $\Delta^i(\boldsymbol{\mathcal{B}})_t^\infty = -\eta^i_t B_{t-1} - \eta^i_t A_{t-1}d_t$. Ricardian equivalence holds and government debt drops

from the aggregate budget constraint. If (gTVC) is not imposed, lump-sum taxes may improve risk sharing.

(b) If taxes are constant or tax shocks are unanticipated, $\mathbb{E}_t[\eta^i_{t+1}] = \eta^i_t$ and $\mathbb{C}ov_t(m^i_{t+1}, \eta^i_{t+1}|z_{t+1}) < 0$, then

$$\lim_{T\to\infty} \mathbb{E}_t \left[m^i_{t|t+T} \eta^i_{t+T} (p_{t+T} B_{t+T} + q_{t+T} A_{t+T} \right] = 0 \tag{tTVC}$$

(c) If $\mathbb{C}ov_t(m^i_{t+1}, \eta^i_{t+1}|z_{t+1}) < 0$ imposing (gTVC) implies (tTVC). However, $\Delta^i(\boldsymbol{\mathcal{B}})^\infty_t > 0$ and government debt still improves risk sharing.

If the relative tax burden is proportional to the share of bonds each household holds, then (iTVC) and (tTVC) are equivalent. In general, neither (gTVC) nor (iTVC) imply that (tTVC) holds.

Proposition 6 clarifies the relation between individual transversality conditions and the present value of interest expense. The reason (gTVC) does not follow from (iTVC) is that the households discount their own future wealth at a higher rate than future surpluses. Part (a) shows that violations of the government transversality condition imply that lump-sum taxes can improve risk-sharing because a bubble component generates a net wealth. To gain intuition, it is useful to isolate the contribution of the present value of interest expense and set $\boldsymbol{g} = 0$ so that (iTVC) implies

$$\sum_{j=0}^{\infty} \mathbb{E}_t[m^i_{t|t+j} c^i_{t+j}] = b^i_{t-1} + a^i_{t-1} d_t + \sum_{j=0}^{\infty} \mathbb{E}_t[m^i_{t|t+j} \phi^i_{t+j} e_{t+j}] + \Delta^i(\boldsymbol{\mathcal{B}})^\infty_t$$

Under the premise of part (a) of Proposition 6, $\lim_{T\to\infty} \Delta^i(\boldsymbol{\mathcal{B}})^T_t = -\eta^i_t B_{t-1} - \eta^i_t A_{t-1} d_t$. Aggregating gives

$$\int_{\mathcal{I}} \sum_{j=0}^{\infty} \mathbb{E}_t[m^i_{t|t+j} c^i_{t+j}] = \int_{\mathcal{I}} \sum_{j=0}^{\infty} \mathbb{E}_t[m^i_{t|t+j} \phi^i_{t+j} e_{t+j}] \tag{15}$$

The present value of aggregate consumption is equal to the present value of aggregate income, and government debt is neutral. If, however, (gTVC) does not hold, then

$$\int_{\mathcal{I}} \sum_{j=0}^{\infty} \mathbb{E}_t[m^i_{t|t+j} c^i_{t+j}] = \int_{\mathcal{I}} \sum_{j=0}^{\infty} \mathbb{E}_t[m^i_{t|t+j} \phi^i_{t+j} e_{t+j}] + \lim_{T\to\infty} \int_{\mathcal{I}} \eta^i_t \mathbb{E}_t \left[m^i_{t|t+T} (p_{t+T} B_{t+T} + q_{t+T} A_{t+T} \right]$$

and government debt does not drop from the aggregate budget constraint. Violations of transversality conditions on government debt drive a wedge between the present value of consumption and the present value of income through $\Delta(\boldsymbol{\mathcal{B}})^\infty_t$. An increase in government debt can make everyone feel wealthier because households discount tax liabilities at a different rate than the safe interest income on their government debt portfolios. However, government debt is more valuable to households that have a higher tax burden today, i.e. $\Delta^i(\boldsymbol{\mathcal{B}})^\infty_t$ is increasing in η^i_t.

Part (b) shows that if households are taxed in proportion to their bond holdings, then the individual transversality condition is sufficient for (tTVC). Brunnermeier et al. (2024) study an economy in which the combination of homothetic preferences and capital taxation imply that the tax burden equals the share of the government debt portfolio held by each household. As such, (iTVC) and (tTVC) are equivalent. I complement their argument by explicitly highlighting the role of fiscal insurance. Market incompleteness implies that a transversality condition on government debt does not hold. However, debt issuance improves risk sharing because of the endogenous response of taxes. If debt issuance increases in periods of high marginal utility, this insurance increases the present value of interest income.

Finally, part (c) shows that, if taxes and pricing kernels are conditionally negatively correlated, a transversality condition on the government portfolio (gTVC) implies that the risk-adjusted present value of interest expense distant in the future goes to zero. The intuition is again that taxes are risky liabilities $\mathbb{C}\text{ov}_t(m^i_{t+1}, \eta^i_{t+1}|z_{t+1}) < 0$, so they are discounted at a higher rate than the entire government portfolio. Imposing a transversality condition on government debt neutralize wealth effect from lump-sum taxes (see Di Tella (2020)) .

4.4 Discussion

I briefly discuss how my paper relates to the literature on safe assets and what would change in government debt was nominal instead or real.

4.4.1 Liquidity Premia and Safe Assets

There are several conceptual differences between the role of government debt in my framework and models in which government debt is special (Krishnamurthy & Vissing-Jorgensen, 2012; Nagel, 2016). A major difference is the channel through which debt supply impacts interest rates. I emphasize the government's liability side. Changes in the supply of government debt affect asset prices because future fiscal adjustments impact households' pricing kernels. However, private and public assets are perfect substitutes and the *convenience yield* on government debt is zero. Still, issuance of private assets is different because of the way in which it is financed. An increase in public debt triggers future tax adjustments, whereas more private debt does not. As a result, issuance of the same asset can have different economic outcomes depending on who the issuer is.

Households do not attribute any special value to government debt per se, but instead value the change in the timing of taxes that helps them better share idiosyncratic risk. The extent to which debt issuance affects asset prices is tied to the covariance between the tax burden and each household's pricing kernel, as it reflects how fiscal policy shapes risk exposures. It is relatively straightforward to introduce liquidity premia, for example through bond-in-the-utility preferences. However, a liquidity premium alone is not sufficient in general to generate real effects (see Di Tella (2020)).

4.4.2 Comparison between Private Debt and Government Debt

Another question is what prevents households from setting up an intermediary that performs the same role as the government. I do not provide an explanation for why such arrangements do not exist within the model. However, in practice, intermediaries may not be able to replicate the fiscal insurance mechanism for several reasons.

First, the government is large and can mandate taxes for everyone in the economy. As a result, the government can raise revenue free of idiosyncratic risk and issue securities that are backed by aggregate cash flows only. An intermediary could perform the same role to the extent that its revenue is also free of idiosyncratic risk. Yet, it seems unlikely that a private intermediary's revenue would remain totally insulated from idiosyncratic shocks from its client base or business operations, preventing it from offering the same risk transformation. Second, private contracts lack the enforcement power of taxation. Households cannot credibly commit to pay a private institution more when they experience positive income shocks.[3] The government's authority over taxation solves this commitment problem by making payments mandatory rather than voluntary.

5 Dynamic Model

The two-period model shows that debt prices vary with debt supply because future tax adjustments alter households' pricing kernels. The sign of the net wealth contribution reveals whether further debt issuance improves risk sharing. While this is useful to understand the relation between fiscal insurance and welfare, the net wealth effect does not generate real effects. Further, discrete time introduces additional complications. In discrete time, idiosyncratic shocks determine income shares in the current period, but the share of interest payments is determined by holdings chosen in the previous period. This timing mismatch complicates the fiscal insurance mechanism. The continuous-time model with capital kills all the other effects except the net wealth contribution while maintaining tractability.

The takeaway is that the impact of fiscal insurance operates very differently in a production economy because it impacts the rate of investment. I then demonstrate that government debt is irrelevant if lump-sum taxes do not impact aggregate outcomes even when markets are incomplete. The second conceptual point is that improvements from risk sharing are linked to the tax side, and that retrading government debt after bad shocks alone does not provide any self-insurance. I articulate the main points in a production economy with idiosyncratic investment risk, no aggregate risk, and log utility. I then discuss extensions with aggregate risk, recursive utility, and uninsurable tax shocks. Even though the stochastic lump-sum plays an important role in the two-period economy, I temporarily abstract from for expositional purposes.

5.1 Steady State Framework

I first study a steady state version of the model without aggregate risk.

[3]A long literature studying limited commitment shows that households' incentive to repay debt declines after good idiosyncratic shocks. See for example Alvarez and Jermann (2000).

5.1.1 Environment

I describe households, government, and define a competitive equilibrium.

Households Time $t \in (0, \infty)$ is continuous and runs from zero to infinity. Boldface letters refer to sequences, e.g., $\boldsymbol{c} = \{c_t\}_{t=0}^{\infty}$. There is a continuum of households indexed by $i \in \mathcal{I}$. All households have identical logarithmic preferences

$$U_t^i(\boldsymbol{c}^i) = \mathbb{E}_t \left[\int_t^{\infty} \log c_s^i \, ds \right] \tag{16}$$

Households own capital k_t^i and produce a flow of consumption goods according to a linear technology

$$e_t^i = (a - \iota_t^i) k_t^i$$

where ι_t^i is the investment rate per unit of capital. Productivity a is common to all households and is constant over time. The change in effective capital over a short period of time is exposed to both aggregate and idiosyncratic shocks

$$\frac{dk_t^i}{k_t^i} = (\Phi(\iota_t^i) - \delta) dt + \nu dZ_t^i$$

where dZ_t^i is an idiosyncratic Brownian motion that is specific to household i. The parameter δ denotes the depreciation rate. The increasing and concave function $\Phi(\iota_t^i)$ captures capital adjustment costs. The volatility of idiosyncratic shocks, ν, is constant. To derive closed-form expressions, I specify the adjustment cost function such that $\Phi(\iota) = \frac{1}{\varrho} \log(1 + \varrho \iota)$ (see Brunnermeier and Sannikov (2014)). Capital is traded continuously at equilibrium price q_t.

Idiosyncratic risk washes out in the aggregate, so that aggregate capital $K_t = \int_{\mathcal{I}} k_t^i \, di$ evolves as

$$dK_t = \int_{\mathcal{I}} dk_t^i \, di = \left(\int_{\mathcal{I}} (\Phi(\iota_t^i) - \delta) k_t^i \, di \right) dt \tag{17}$$

Government I consider a government that raises taxes and issues real risk-free debt to fund an exogenous stream of government purchases $\boldsymbol{g}$. The government budget constraint is

$$d\mathcal{B}_t = \mathcal{B}_t r_t \, dt - \tau_t \, dt + g_t \, dt \tag{18}$$

where $d\mathcal{B}_t = \mu_t \mathcal{B}_t \, dt$ denotes bond issuance, $\mathcal{B}_t r_t \, dt$ is interest expense, $\tau_t \, dt$ is aggregate taxation, and g_t is government purchases. Government policy is *exogenously* given by a stochastic process for debt issuance μ_t that loads on the same aggregate Brownian shock as capital. In the steady state, I assume that the issuance rate is constant and equal to μ. For simplicity, government purchases are a constant fraction of aggregate capital $g_t = gK_t$. *Aggregate* taxation adjusts so that the government budget constraint holds in every period. As in the two-period model, I restrict attention to an affine tax schedule. The government collects a fraction $\kappa \in [0, 1]$ from lump-sum taxes and a fraction $(1 - \kappa)$ from proportional taxes. I assume that proportional taxes are collected on financial wealth n_t^i, which is the sum of government debt and capital held by each agent . As a result, total tax revenue is

$$\tau_t = \tau_t^l + \tau_t^n \int_{\mathcal{I}} n_t^i di$$

where tax rates on each instrument adjust as follows

$$\tau_t^l = \kappa \tau_t \qquad : \qquad \tau_t^n \int_{\mathcal{I}} n_t^i di = (1-\kappa)\tau_t$$

The lump-sum tax is a pure lump-sum tax that is identical for everyone. If $\kappa = 1$, the tax burden is constant over time across all households. The proportional tax implies that the tax burden falls more on households that hold more capital. Because capital accumulation depends on idiosyncratic risks, this is what creates uninsurable variation in the tax burden across households.

5.1.2 Households' Problem

As in the two-period model, I define households' wealth w_t^i as the market value of financial assets less the present value of their individual tax liabilities.[1]Hence, net wealth is

$$w_t^i = q_t k_t^i + b_t^i - T_t^i$$

Individual tax liabilities vary with idiosyncratic shocks. As a result, I compute the present value T_t^i using households' individual pricing kernel ξ_t^i. The pricing kernel evolves as

$$\frac{d\xi_t^i}{\xi_t^i} = -r_t \, dt - \pi_t \, dZ_t^i$$

where r_t is the equilibrium risk-free rate and π_t is households' exposure to idiosyncratic shocks. As a result, the present value of taxes is

$$T_t^i = \mathbb{E}_t \left[\int_t^\infty \frac{\xi_s^i}{\xi_t^i} \tau_s^i \, ds \right]$$

Since T_t^i is an Itô process that varies with idiosyncratic shocks, I postulate that

$$dT_t^i = \mu_t^T T_t^i \, dt + \sigma_t^T T_t^i \, dZ_t^i - \tau_t^i \, dt$$

where μ_t^T and σ_t^T denote the drift and diffusion terms.

As in Di Tella (2020), I assume that markets are incomplete in the sense that idiosyncratic risk cannot be shared. There are no other financial frictions. The return on capital is

$$dR_t^i(\iota) = \frac{a - \iota_t^i}{q_t} \, dt + \frac{d(q_t k_t^i)}{q_t k_t^i} = \mathbb{E}[dR_t^i] \, dt + \nu \, dZ_t^i$$

[1]This definition of wealth follows from solving the dynamic budget constraint forward and imposing a no-ponzi condition. Appendix A.3.2 explains this relation in more detail.

where $\mathbb{E}[dR_t^i]$ includes the dividend yield and the capital gain. The households' problem is to choose consumption c_t^i, investment ι_t^i, capital holdings k_t^i, and bond holdings b_t^i to maximize (20) subject to

$$dw_t^i = b_t^i r_t \, dt + q_t k_t^i \mathbb{E}[dR_t^i] \, dt - c_t^i \, dt - \tau_t^i \, dt + q_t k_t^i \nu \, dZ_t^i \tag{19}$$

a solvency constraint $w_t^i \geqslant 0$ and a no-ponzi condition $\lim_{t \to \infty} \mathbb{E}[\xi_t^i w_t^i] = 0$. Including the present value of taxes in the definition of wealth, implicitly impose a natural borrowing limit.[2] I denote the wealth share invested in capital as

$$\varphi_t^{k,i} = \frac{q_t k_t^i}{w_t^i}$$

In most representative agent asset pricing models, the wealth share of capital $\varphi_t^{k,i}$ equals one in equilibrium because capital is the sole source of wealth and financial assets are in zero net supply. The wealth share of capital also equals one in representative agent economies with a government because the present value of taxes and the market value of total bond positions usually coincide. The key novelty of this framework is that $\varphi_t^{k,i}$ may be different than one.

5.1.3 Competitive Equilibrium

A competitive equilibrium, given *exogenous* government policy μ_t is defined in the usual way as a set of allocations and prices such that all households maximize utility and all markets clear. I take the initial capital stock k_0 and its distribution among house as given. All households are endowed with $k_t^i > 0$, so that everyone starts with strictly positive net worth.

Definition 3 (Competitive Equilibrium). *A competitive equilibrium with taxes is a set of aggregate stochastic processes: the price of capital q_t, the aggregate capital stock k_t, taxes τ_t^l, τ_t^n and a set of stochastic processes for each household $i \in \mathcal{I}$: net worth w_t^i, consumption c_t^i, portfolios θ_t^i, and investment ι_t^i such that*

(i) each household chooses c_t^i, ι_t^i, θ_t^i to maximize utility (20) subject to the budget constraint (19) taking aggregates as given and for arbitrary initial net worth $w_0^i > 0$;

(ii) given issuance μ_t, capital taxes τ_t^k and lump-sum taxes τ_t^l satisfy the government budget constraint (18);

(iii) aggregate capital is consistent with the initial condition k_0 and satisfies the law of motion (21);

(iv) all markets clear

$$\int_{\mathcal{I}} c_t^i di + \mathfrak{g} K_t + \int_{\mathcal{I}} \iota_t^i k_t^i di = a K_t \tag{Goods}$$

$$\int_{\mathcal{I}} \varphi_t^{k,i} w_t^i di = q_t K_t \tag{Capital}$$

The bond market clears by Walras' Law. The total value of bond positions adds up to $\mathcal{B}_t$.

[2] Alternatively, a borrowing constraint that varies with the present value of taxes would also work.

5.1.4 Model Summary

Figure 8 provides a visual summary of the main ingredients of the model. The government issues debt to a continuum of households who operate risky capital. The key ingredient of the model is that tax liabilities reflect uninsurable risks. In contrast, the market value of government debt reflect the present value of aggregate tax revenue, which is free of idiosyncratic risks.

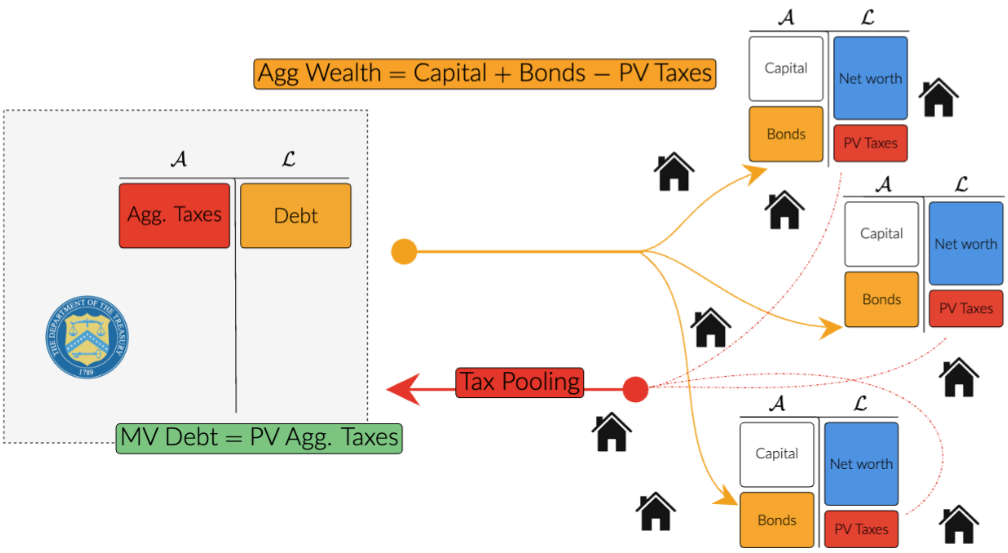


Figure 8: Architecture and summary of continuous-time model with production. The government issues debt to a continuum of households subject to uninsurable idiosyncratic investment shocks. Government debt is backed by aggregate tax revenue τ_t that is free of idiosyncratic risk. In contrast, if taxes partly depend on the history of idiosyncratic shocks, individual taxes reflect uninsurable shocks. Fiscal insurance implies that the aggregate individual valuation of future tax liabilities, i.e., the aggregate value of the red blocks on the right, $\int_{\mathcal{I}} T_t^i, di$, need not coincide with the market value of government bonds given by the yellow blocks.

5.2 Solving for the Equilibrium

I first describe households' optimality conditions and show that everyone makes the same consumption, savings, and investment decisions. I then solve for aggregate quantities in terms of the total share of wealth invested in capital. To isolate the impact of fiscal insurance, I set $\mathfrak{g} = 0$.

5.2.1 Households' Optimality

Households choose consumption c_t^i, the portfolio share of capital $\varphi_t^{k,i}$, and the rate of investment ι_t^i. Lemma 3 demonstrates that optimal policies are linear in net wealth w_t^i and that all households make the same choices. This property considerably simplifies aggregation, as I do not need to keep track of the entire wealth distribution across households.

Lemma 3. *Households' consumption and investment policies satisfy*

$$c_t^i = \rho w_t^i,$$
$$\Phi'(\iota_t^i) = \frac{1}{q_t}.$$

Further, the share of wealth invested in capital satisfies

$$\mathbb{E}[dR_t^i(\iota_t^i)] - r_t = \nu^2 \varphi_t^{k,i} - \nu \sigma_t^T \varphi_t^{T,i}$$

where $\varphi_t^{T,i} = \frac{T_t^i}{w_t^i}$ *is the share of tax liabilities in total wealth, such that* $1 = \varphi_t^{k,i} + \varphi_t^{B,i} - \varphi_t^{T,i}$.

An interesting implication of Lemma 3 is that all households make the same choices even when taxes are lump-sum. The standard intuition with uninsurable income risk is that lump-sum taxes are more burdensome for unlucky households, as they represent a larger share of wealth for those who experienced a series of negative income shocks. This intuition no longer holds in a model with idiosyncratic investment risks where policies are linear in net wealth w_t^i.

The risk premium on capital consists of two components. The first component $\nu^2 \varphi_t^{k,i}$ reflects a compensation for uninsurable risk that households cannot diversify. The second component $\nu \sigma_t^T \varphi_t^{T,i}$ reflects fiscal insurance. A decline in the present value of taxes after a bad idiosyncratic shock lowers the present value of tax liabilities. Hence, depending on its loading on idiosyncratic shocks σ_t^T, changes in the present value of taxes provide insurance against idiosyncratic shocks.[3]

5.2.2 The Wealth Share of Capital and Aggregate Quantities

Given the combination of homothetic preferences and linear policies, it turns out that all aggregate equilibrium quantities can be written in terms of the aggregate share of net wealth allocated to capital[4]

$$\varphi_t^k = \frac{q_t K_t}{\int_{\mathcal{I}} w_t^i di} = \frac{q_t K_t}{W_t}$$

I look for a symmetric stationary equilibrium. Because there is no aggregate risk, the price of capital q_t is constant. Proposition 7 characterizes the equilibrium price of capital and the investment rate in terms of the capital wealth share φ_t^k and other exogenous parameters. I also provide an expression for the risk-free rate in terms of φ_t^k, φ_t^T, and the loading of taxes σ_t^T on idiosyncratic shocks. The results follow immediately by aggregating individual demand and clearing the capital and goods market.

[3]The fact that the present value of tax liabilities loads on the idiosyncratic shock dZ_t^i is a major deviation from Di Tella (2020). When money and transfers are distributed lump-sum, there is no hedging term that affects portfolio demand.

[4]When there is a government collecting taxes from households, it is useful to distinguish φ_t^k from the share of total assets allocated to capital ϑ_t^k. These are distinct objects because the denominator in ϑ_t^k includes only tradable financial assets, while φ_t^k divides by net wealth. Silva (2020) makes a similar point in the context of QE interventions.

Proposition 7 (Aggregate Outcomes)**.** *The equilibrium investment rate* $\iota_t^i = \iota_t$ *and the price of capital* q_t *are*

$$\iota_t = \frac{a\varphi_t^k - \rho}{\rho\varrho + \varphi_t^k}$$
$$q_t = 1 + \varrho\left(\frac{a\varphi_t^k - \rho}{\rho\varrho + \varphi_t^k}\right)$$

The risk-free rate is

$$r_t = \frac{a - \iota_t}{q_t} + \Phi(\iota_t) - \delta + \nu^2\varphi_t^k + \nu\sigma_t^T\varphi_t^T$$

The investment rate depends on fiscal policy only through φ_t^k. As a result, changes in the rate of issuance of government debt will have real effects only to the extent that they impact φ_t^k. The continuous-time framework clearly isolates the fiscal insurance channel.
The last step in the equilibrium characterization is to demonstrate that $\varphi_t^k \neq 1$ and to show whether it depends on the tax structure. To this purpose, I postulate a stochastic process for the tax burden $\eta_t^i > 0$.[5] The relative tax burden evolves as

$$\frac{d\eta_t^i}{\eta_t^i} = \sigma_t^\eta dZ_t^i$$

where the loading σ_t^η depends on the tax structure. In the special case of lump-sum taxes, $\sigma_t^\eta = 0$. Importantly, the process $\frac{d\eta_t^i}{\eta_t^i}$ is a martingale, so that all shocks to relative tax burdens are unanticipated. Proposition 8 provides conditions under which the capital share of wealth is different from one. To avoid confusion, I impose a transversality condition on government debt.

Proposition 8 (Capital Wealth Share and Tax Burden)**.** *The wealth share of taxes* φ_t^T *satisfies*

$$\varphi_t^T = \varphi_t^b \int_t^\infty \exp\left(-\int_t^s r_u du - \int_t^s \pi_s\sigma_u^\eta ds + \int_t^s \mu_s ds\right)(r_s - \mu_s)ds$$

assuming that a transversality condition on government debt holds. In a stationary equilibrium, φ_t^T *is*

$$\varphi_t^T = \varphi_t^b \frac{r_t - \mu_t}{r_t + \pi_t\sigma_t^\eta - \mu_t}$$

If either (i) markets are complete or (ii) the relative tax burden is constant $\sigma_t^\eta = 0$*, then* $\varphi_t^k = 1$ *and aggregate outcomes are independent of the debt issuance rate* μ*.*

Proposition 8 is the continuous-time counterpart of Proposition 2. To understand this result better, recall that the wealth shares φ_t^k and φ_t^T satisfy the accounting identity

$$\varphi_t^k + \varphi_t^b = 1 + \varphi_t^T$$

where $\varphi_t^b = \mathcal{B}_t/W_t$ is the share of government debt relative to aggregate wealth $W_t = \int_\mathcal{I} w_t^i di$. Typically, $\varphi_t^T < \varphi_t^b$, so that households' equilibrium exposure to capital is less than one.

[5] This is the continuous-time counterpart of $\eta_1(z, y^i)$ in the two-period model. Most of the previous arguments go through in the dynamic model as well.

Abstracting from government purchases, standard asset pricing models imply that the market value of government debt equals the present value of aggregate tax revenue[6]. As a result, $\varphi_t^b = \varphi_t^T$ must hold so that $\varphi_t^k = 1$ regardless of the growth rate of debt. However, when markets are incomplete, the present value of aggregate taxation need not equal the aggregate present value of individual tax liabilities, so that $\varphi_t^b \neq \varphi_t^T$. Working in continuous time provides a precise way of characterizing the wedge $\Delta(\mathcal{B}) = \varphi_t^b - \varphi_t^T$. The dynamic model ties back to and generalizes the insights from the two-period model about the covariance between pricing kernels and tax burdens. However, because it affects the rate of investment, the net wealth effect $\Delta(\mathcal{B}) = \varphi_t^b - \varphi_t^T$ has real effects.

5.3 Results

I establish two main results. First, I show that lump-sum taxes are neutral despite market incompleteness. This result shows that government debt alone does not improve risk sharing by allowing households to sell bonds in exchange for consumption goods after adverse idiosyncratic shocks. Second, I show that capital taxation lowers investment and thus economic growth. Importantly, the impact of government debt on asset prices does not work entirely through precautionary savings, but also through an intertemporal smoothing motive because of lower consumption growth.

5.3.1 Neutrality of Lump-sum Taxes in Incomplete Markets

Consider the special case of lump-sum taxation, so that $\kappa = 1$. Since there is no aggregate risk and lump-sum taxes are shared equally, the path of lump-sum taxes is deterministic. The present value of all future lump-sum taxes is thus

$$T_t^i = \int_t^\infty e^{-\int_t^s r_u du} \tau_s^i ds$$

It immediately follows that

$$dT_t^i = r_t T_t^i dt - \tau_t^i dt$$

so that $\sigma_t^T = 0$. Proposition 9 shows that if taxes are lump-sum, there is no fiscal insurance. As a result, a version of Ricardian equivalence holds even though markets are incomplete. Government debt alone cannot improve risk sharing if the relative tax burden is constant across households.

Proposition 9 (Lump-sum Taxes)**.** *If taxation is lump-sum $\kappa = 1$, then fiscal policy is irrelevant for aggregate outcomes. Investment and the price of capital are independent of the growth rate of debt μ.*

Proposition 9 delivers a surprising version of Ricardian neutrality with incomplete markets. Government debt is neutral even in an economy with uninsurable risks. This revisits the conventional wisdom that government debt improves risk sharing because households can exchange bonds for

[6]This follows from solving the government budget constraint forward. I derive this relation in Appendix A.5.

goods after a series of bad idiosyncratic shocks. The wealth share of capital is $\varphi_t^k = 1$ so that

$$q_t = \frac{a}{\rho}$$
$$\iota_t = \frac{a-\rho}{\rho}$$

only depend on how productive and patient households are.

5.3.2 The Impact of Fiscal Insurance on Investments

Suppose the government only collects capital taxes, so that $\kappa = 0$. Then, the present discounted value of future tax liabilities is

$$T_t^i = \mathbb{E}_t \left[\int_t^\infty \frac{\xi_s^i}{\xi_t^i} q_s k_s^i ds \right]$$

Proposition 10 establishes that taxes on capital provide fiscal insurance. Specifically, the tax burden rises after a good idiosyncratic shock, so that $\sigma_t^\eta > 0$ and $\varphi_t^k < 1$.

Proposition 10 (Fiscal Insurance)**.** *If the government raises taxes on capital, $\sigma_t^\eta > 0$ and $\pi_t > 0$. As a result,*

$$\varphi_t^T = \varphi_t^b \frac{r_t - \mu_t}{r_t + \pi_t \sigma_t^\eta - \mu_t} < \varphi_t^b$$

The wealth share of capital is less than one $\varphi_t^k < 1$.

In an economy with capital taxes, individual tax liabilities depend on the history of idiosyncratic shocks. The combination of homothetic preferences and linear policies implies that lucky households hold a larger share of capital. Thus, the tax burden increases after a series of good shocks. Households' individual tax liabilities are effectively a risky cash flow, and thus get discounted at a higher rate that captures uninsurable idiosyncratic risk that households have to bear in equilibrium. Corollary 3, which immediately follows by differentiating ι with respect to φ_t^k, establishes that investment increases with the wealth share of capital. As a result, fiscal insurance reduces capital accumulation. Importantly, this result does not depend on the specific tax instruments in the model. A consumption tax would also generate a similar effect, on top of directly distorting the consumption margin.

Corollary 3. *The investment rate ι_t increases as the share of wealth invested in capital increases.*

$$\frac{\partial \iota}{\partial \varphi_t^k} > 0$$

Corollary 3 reveals that the net wealth effect of government debt leads to real effects in the production economy. Because taxes are discounted at a higher rate, households feel richer and want to consume more. The only way to accommodate higher consumption is for investment to decline. This mechanism explains why the net wealth contribution of government debt does not impact asset prices in the endowment economy of Section 3. Households do feel richer, but how much they consume is determined by the exogenous endowment in period $t = 0$.

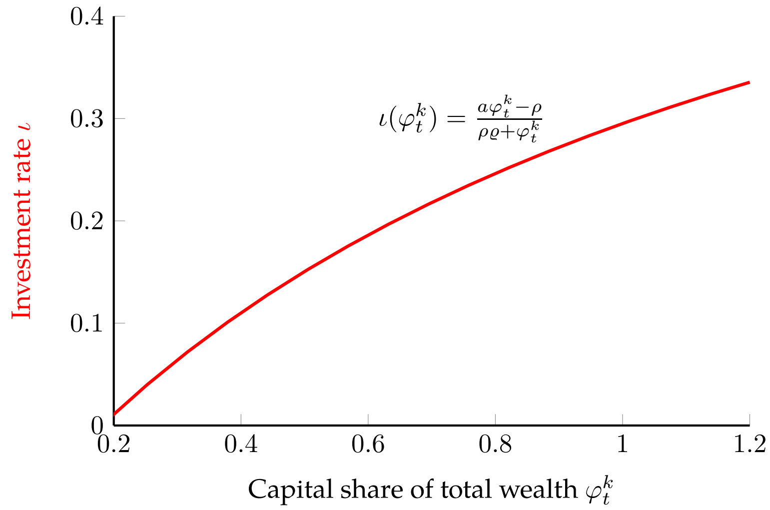


Figure 9: Investment rate as a function of the capital wealth share φ_t^k

Before concluding, I briefly discuss extensions to the baseline framework. The mechanism in Proposition 8 is robust to the introduction of aggregate risk, recursive utility, imperfect risk sharing, and idiosyncratic tax risk. I only emphasize novel implications.

5.3.3 Recursive Utility and Supply Risks

Ingredients: Suppose all households have preferences defined recursively (see Duffie and Epstein (1992)) by

$$U_t^i(\boldsymbol{c}^i) = \mathbb{E}_t\left[\int_t^\infty f(c_s^i, U_s^i)ds\right] \tag{20}$$

where

$$f(c,U) = \frac{1}{1-\psi}\left\{\frac{\rho c^{1-\psi}}{[(1-\gamma)U]^{(\gamma-\psi)/(1-\gamma)}} - \rho(1-\gamma)U\right\}$$

All households share the same discount rate $\rho > 0$, risk aversion γ, and elasticity of intertemporal substitution (IES) ψ^{-1}. Standard CRRA utility is nested as the special case $\gamma = \psi$. In addition of decoupling risk aversion and IES, there are two key features of preferences (20). The first is that policies are linear in wealth, which considerably simplifies aggregation. Second, when $\gamma > 1$, these preferences generate intertemporal hedging demand.

The change in capital over a short period of time is exposed to both aggregate and idiosyncratic shocks

$$\frac{dk_t^i}{k_t^i} = (\Phi(\iota_t^i) - \delta)dt + \sigma dZ_t + \nu dZ_t^i$$

where $Z = \{Z_t \in \mathbb{R}; \mathcal{F}_t, t \geqslant 0\}$ is an aggregate Brownian motion and $Z^i = \{W_t^i, \mathcal{F}_t, t \geqslant 0\}$ is an idiosyncratic Brownian motion that is specific to household i.

Idiosyncratic risk washes out in the aggregate, so that aggregate capital $k_t = \int_{\mathcal{I}} k_t^i di$ evolves

$$dK_t = \int_{\mathcal{I}} dk_t^i di = \left(\int_{\mathcal{I}} (\Phi(\iota_t^i) - \delta) k_t^i di \right) + \sigma k_t dZ_t \tag{21}$$

Government behavior is the same as above, with the only difference that the issuance rate is stochastic.

$$\mu_t ...$$

Implications: The price of capital q_t is no longer constant follows a geometric Brownian motion

$$\frac{dq_t}{q_t} = \mu_{q,t} dt + \sigma_{q,t} dZ_t$$

The return on household's i capital is this given by

$$dR_t^i = \frac{a - \iota_t^i}{q_t} dt + \left(\mu_{q,t} + \Phi(\iota_t^i) - \delta + \sigma_{q,t}\sigma \right) dt + (\sigma_{q,t} + \sigma) dZ_t + \nu dZ_t^i$$

Markets for aggregate shocks are complete. There exists a *market* pricing kernel $\overline{\xi}_t$ such that

$$\frac{d\overline{\xi}_t}{\overline{\xi}_t} = -r_t dt - \pi_t dZ_t$$

However, the market SDF ξ_t does not correspond to any households' marginal utility of wealth because the latter also compensates exposure to tax shocks and idiosyncratic capital shocks. Such differences reflect absence of full risk-sharing, for which I provide more details below.

5.3.4 Partial Risk Sharing

Ingredients: I now allow households to share part of their idiosyncratic risk by issuing equity. Households I impose imperfect risk sharing in financial markets by requiring household i to maintain a fixed exogenous exposure χ to his own return on capital, dR_t^i.[7] The contractual environment is therefore close to Di Tella (2017), but all households are equally productive. There are no other financial frictions. In particular, households are free to short capital and government debt. Households can also write contracts, referred to as hedging positions (see Hansen, Khorrami, and Tourre (2024)), contingent on the aggregate state Z_t. The return on these hedging positions is dR_t^θ, which also only reflects exposure to aggregate Brownian risk. Constraints on idiosyncratic risk sharing do not prevent the household to perfectly share aggregate risk.

Capital is traded continuously at equilibrium price q_t. Households can issue outside equity to diversify away part of their idiosyncratic risk, but must retain a fraction of it. The households' problem is to

[7]There are different ways to microfound this assumption, for example by invoking agency frictions Di Tella (2017). A portfolio constraint on equity issuance is similar if it always binds in equilibrium, e.g. Brunnermeier et al. (2024).

maximize expected utility subject to the dynamic budget constraint

$$\frac{dw_t^i}{w_t^i} = \varphi_t^b r_t dt + \varphi_t^{k,i}(dR_t^i - r_t dt) - (1-\chi)\theta_t^{K,i}(dR_t - r_t dt) \\ + \theta_t^{c,i}(dR_t^\theta - r_t dt) - c_t^i dt - d\text{PV(taxes)}$$

Implications: Issuance of outside equity attenuates the effects of fiscal insurance as it lowers households' equilibrium exposure to uninsurable risk. Proposition 11 summarizes this result.

Proposition 11 (Partial Risk Sharing). *Households' optimality for consumption and investment requires*

$$c_t^i = \rho^{1/\psi}\zeta^{(\psi-1)/\psi} w_t^i$$
$$q_t = \frac{1}{\Phi'(\iota_t)}$$

where ζ captures time-varying investment opportunities. All households choose the same exposure to idiosyncratic shocks $\varphi_t^{k,i} = \varphi_t^k$. Expected excess return on capital are

$$\frac{a-\iota_t^i}{q_t} + \Phi(\iota_t^i) - \delta + \mu_{q,t} + \sigma_{q,t}\sigma - r_t = \pi_t(\sigma_{q,t}+\sigma) + \gamma\varphi_t^k\chi^2\nu^2 - \nu\sigma_t^T\varphi_t^T$$

5.3.5 Idiosyncratic Tax Shocks

Ingredients: Lastly, I introduce idiosyncratic shocks to taxation. In the spirit of Achdou, Han, Lasry, Lions, and Moll (2021), I assume that the households' wealth tax rate evolves as a continuous time Markov chain that takes two values $\tau_t^i \in \{\tau_l, \tau_h\}$ where $\tau_h > \tau_l$ corresponds to a high-taxation regime. Tax process associated to each household are independent, and the instantaneous transition probabilities between the two states are $\lambda_{h\to l}$ and $\lambda_{l\to h}$, respectively.

Implications: In the model, the distribution of wealth between high- and low-tax households emerges as a key state variable. Households exhibit heterogeneous marginal propensities to consume due to differences in investment opportunities, and shocks to government debt issuance tend to redistribute wealth from low- to high-tax households. These differences in portfolio choices across households also lead to a concentration of risk, amplifying the heterogeneity in consumption and wealth dynamics. Overall, the interaction of wealth distribution, investment opportunities, and debt shocks shapes both risk sharing and the allocation of resources.

6 Conclusion

I model the impact of government debt on bond prices when fiscal policy interacts with uninsurable risks. The main message of the paper is that changes in debt supply trigger future tax adjustments that impact households' risk exposure, and thus their pricing kernels. Contrary to conventional asset pricing models, the pricing implications of government debt issuance originate from its liability side and how it is structured.

The two-period framework shows that in an endowment economy, households' exposure to uninsurable risks scales with the level of debt. An increase in debt raises interest rates because higher taxation in the future redistributes risk exposures across households and lowers their consumption volatility. The covariance between the individual pricing kernels and the relative tax burden determines how an increase in debt impacts the price of risk-free and risky assets. Fiscal insurance implies that government debt is net wealth. However, this wealth effect is irrelevant in an endowment economy as the current level of consumption is determined by the endowment realization. Still, the sign of the net wealth contribution signals whether further debt issuance leads to welfare gains.

The relation between bond prices and bond supply is non-monotonic. I find that there exists a debt threshold after which further debt issuance exposes households to fiscal risks. This debt threshold is lower when taxation is less redistributive or when foreign investors are less price elastic. A promising avenue for further research is a quantitative exercise that assesses how far various countries are from this threshold. In the production economy, the wealth effect of fiscal insurance has real effects because households now have access to a storage technology. Working in continuous time allows me to isolate this channel. With homothetic preferences and linear policies, households always hold government bonds in proportion to the taxes they pay. As a result, any gains comes purely through valuation effects. Because taxes are discounted at a higher rate than government debt, households feel wealthier. As a result, they consume more and invest less.

I show that a version of Ricardian equivalence holds even under market incompleteness. Households can entirely undo lump-sum taxes through private trading, as they have enough assets to do so and are not subject to borrowing constraints except the natural borrowing limit. Beyond establishing a novel benchmark result, this result delivers an important conceptual point. Government debt alone does not improve risk sharing. Crucially, it is the structure of taxation that determines whether government debt is net wealth.

References

Achdou, Y., Han, J., Lasry, J.-M., Lions, P.-L., & Moll, B. (2021). Income and wealth distribution in macroeconomics: A continuous-time approach. *The Review of Economic Studies*, *89*(1), 45–86.

Aiyagari, S., Marcet, A., Sargent, T., & Seppälä, J. (2002). Optimal taxation without state-contingent debt. *Journal of Political Economy*, *110*(6), 1220–1254.

Aiyagari, S., & McGrattan, E. R. (1998). The optimum quantity of debt. *Journal of Monetary Economics*, *42*(3), 447–469.

Alvarez, F., & Jermann, U. J. (2000). Efficiency, equilibrium, and asset pricing with risk of default. *Econometrica*, *68*(4), 775–797.

Angeletos, G.-M. (2002). Fiscal policy with noncontingent debt and the optimal maturity structure. *The Quarterly Journal of Economics*, *117*(3), 1105–1131.

Angeletos, G.-M. (2007). Uninsured idiosyncratic investment risk and aggregate saving. *Review of Economic Dynamics*, *10*(1), 1–30.

Auclert, A., Rognlie, M., & Straub, L. (2024). *Fiscal and monetary policy with heterogeneous agents* (Working Paper No. 32991).

Barro, R. J. (1974). Are government bonds net wealth? *Journal of Political Economy*, *82*(6), 1095–1117.

Barsky, R. B., Mankiw, N. G., & Zeldes, S. P. (1986). Ricardian consumers with keynesian propensities. *The American Economic Review*, *76*(4), 676–691.

Bassetto, M., & Kocherlakota, N. (2004). On the irrelevance of government debt when taxes are distortionary. *Journal of Monetary Economics*, *51*(2), 299–304.

Bewley, T. (1979). *The optimum quantity of money* (Discussion Paper No. 383). Retrieved from https://hdl.handle.net/10419/220743

Bhandari, A., Evans, D., Golosov, M., & Sargent, T. J. (2017a). Fiscal policy and debt management with incomplete markets. *The Quarterly Journal of Economics*, *132*(2), 617–663.

Bhandari, A., Evans, D., Golosov, M., & Sargent, T. J. (2017b). Public debt in economies with heterogeneous agents. *Journal of Monetary Economics*, *91*, 39–51.

Bianchi, F., Dabla-Norris, E., & Khalid, S. (2025). *Perceptions of public debt and policy expectations: Evidence from cross country surveys* (Working Paper No. 34382). National Bureau of Economic Research.

Brunnermeier, M. K., Merkel, S., & Sannikov, Y. (2024). Safe assets. *Journal of Political Economy*, *132*(11), 3603–3657.

Brunnermeier, M. K., & Sannikov, Y. (2014). A macroeconomic model with a financial sector. *American Economic Review*, *104*(2), 379-–421.

Chan, L. K. C. (1983). Uncertainty and the neutrality of government financing policy. *Journal of Monetary Economics*, *11*(3), 351–372.

Constantinides, G. M., & Duffie, D. (1996). Asset pricing with heterogeneous consumers. *Journal of Political Economy*, *104*(2), 219–240.

d'Avernas, A., Hubert de Fraisse, A., Ning, L., & Vandeweyer, Q. (2024). *The fiscal cost of quantitative* *easing* (Working Paper).

d'Avernas, A., & Vandeweyer, Q. (2024). Treasury bill shortages and the pricing of short-term assets. *The Journal of Finance*, *79*(6), 4083–4141.

Di Tella, S. (2017). Uncertainty shocks and balance sheet recessions. *Journal of Political Economy*, *125*(6), 2038–2081.

Di Tella, S. (2020). Risk premia and the real effects of money. *American Economic Review*, *110*(7), 1995—2040.

Drechsler, I., Savov, A., & Schnabl, P. (2018). A model of monetary policy and risk premia. *The Journal of Finance*, *73*(1), 317–373.

Duffie, D. (2001). *Dynamic asset pricing theory* (3rd ed.). Princeton, NJ: Princeton University Press.

Duffie, D., & Epstein, L. G. (1992). Stochastic differential utility. *Econometrica*, *60*(2), 353–394.

Dávila, J., Hong, J. H., Krusell, P., & Ríos-Rull, J.-V. (2012). Constrained efficiency in the neoclassical growth model with uninsurable idiosyncratic shocks. *Econometrica*, *80*(6), 2431–2467.

D'Amico, S., & King, T. B. (2013). Flow and stock effects of large-scale treasury purchases: Evidence on the importance of local supply. *Journal of Financial Economics*, *108*(2), 425-448.

Feenberg, D., & Coutts, E. (1993). An introduction to the taxsim model. *Journal of Policy Analysis and Management*, *12*(1), 189–194.

Golosov, M., Kocherlakota, N., & Tsyvinski, A. (2003). Optimal indirect and capital taxation. *The Review of Economic Studies*, *70*(3), 569–587.

Gomez Cram, R., Kung, H., Lustig, H., & Zeke, D. (2025). *Fiscal redistribution risk in treasury markets* (Working Paper No. 33769). National Bureau of Economic Research.

Greenwood, R., Hanson, S., & Stein, J. C. (2010). A gap-filling theory of corporate debt maturity choice. *The Journal of Finance*, *65*(3), 993–1028.

Greenwood, R., Hanson, S. G., & Stein, J. C. (2015). A comparative-advantage approach to government debt maturity. *The Journal of Finance*, *70*(4), 1683–1722.

Greenwood, R., & Vayanos, D. (2014). Bond supply and excess bond returns. *The Review of Financial Studies*, *27*(3), 663–713.

Haddad, V., Moreira, A., & Muir, T. (2025). *Asset purchase rules: How qe transformed the bond market* (Working Paper).

Hansen, L. P., Khorrami, P., & Tourre, F. (2024). Comparative valuation dynamics in production economies: Long-run uncertainty, heterogeneity, and market frictions. *Annual Review of Financial Economics*, *16*(Volume 16, 2024), 1–38.

Heathcote, J., Storesletten, K., & Violante, G. L. (2017). Optimal tax progressivity: An analytical framework. *The Quarterly Journal of Economics*, *132*(4), 1693–1754.

Holmström, B., & Tirole, J. (1998). Private and public supply of liquidity. *Journal of Political Economy*, *106*(1), 1–40.

Huggett, M. (1993). The risk-free rate in heterogeneous-agent incomplete-insurance economies. *Journal of Economic Dynamics and Control*, *17*(5), 953–969.

Jiang, Z., Lustig, H., Van Nieuwerburgh, S., & Xiaolan, M. Z. (2023). Fiscal capacity: An asset pricing perspective. *Annual Review of Financial Economics*, *15*(Volume 15, 2023), 197–219.

Jiang, Z., Lustig, H., Van Nieuwerburgh, S., & Xiaolan, M. Z. (2024). The u.s. public debt valuation puzzle. *Econometrica*, *92*(4), 1309–1347.

Kaplan, G., Nikolakoudis, G., & Violante, G. L. (2023). *Price level and inflation dynamics in heterogeneous agent economies* (NBER Working Paper No. 31433).

Kimball, M. S. (1990). Precautionary saving in the small and in the large. *Econometrica*, *58*(1), 53–73.

Kimball, M. S., & Mankiw, N. G. (1989). Precautionary saving and the timing of taxes. *Journal of Political Economy*, *97*(4), 863–879.

Kocherlakota, N. R. (2005). Zero expected wealth taxes: A mirrlees approach to dynamic optimal taxation. *Econometrica*, *73*(5), 1587–1621.

Krishnamurthy, A., & Li, W. (2022). The demand for money, near-money, and treasury bonds. *The Review of Financial Studies*, *36*(5), 2091–2130.

Krishnamurthy, A., & Vissing-Jorgensen, A. (2011). The effects of quantitative easing on interest rates: Channels and implications for policy. *Brookings Papers on Economic Activity*, *2011*(2), 215–287.

Krishnamurthy, A., & Vissing-Jorgensen, A. (2012). The aggregate demand for treasury debt. *Journal of Political Economy*, *120*(2), 233–267.

Krueger, D., & Lustig, H. (2010). When is market incompleteness irrelevant for the price of aggregate risk (and when is it not)? *Journal of Economic Theory*, *145*(1), 1–41.

Lucas, R. E., & Stokey, N. L. (1983). Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics*, *12*(1), 55–93.

Mankiw, G. (2025). *The fiscal future* (17th Annual Feldstein Lecture).

Nagel, S. (2016). The liquidity premium of near-money assets*. *The Quarterly Journal of Economics*, *131*(4), 1927–1971.

Ray, W., Droste, M., & Gorodnichenko, Y. (2024). Unbundling quantitative easing: Taking a cue from treasury auctions. *Journal of Political Economy*, *132*(9), 3115–3172.

Scheinkman, J. A., & Weiss, L. (1986). Borrowing constraints and aggregate economic activity. *Econometrica*, *54*(1), 23–45.

Silva, D. (2020). *The risk channel of unconventional monetary policy* (Working Paper).

Tabova, A. M., & Warnock, F. E. (2024). *Preferred habitats and timing in the world's safe asset* (Working Paper No. 30722). National Bureau of Economic Research.

Tobin, J. (1971). *Essays in economics, volume 1: Macroeconomics*. Cambridge, MA: MIT Press.

Vayanos, D., & Vila, J.-L. (2021). A preferred-habitat model of the term structure of interest rates. *Econometrica*, *89*(1), 77–112.

Weigand, C. (2025). *The effect of fiscal policy shocks on asset prices* (Working Paper).

Werning, I. (2007). Optimal fiscal policy with redistribution. *The Quarterly Journal of Economics*, *122*(3), 925–967.

Appendices

A Proofs

A.1 Proofs for Section 3

A.1.1 Proof of Lemma 1

Proof. Households are ex ante identical and maximize the same concave objective subject to linear budget constraints. Symmetry of equilibrium follows immediately. ■

A.1.2 Proof of Proposition 1

Proof. Lemma 1 implies that c_0^i is independent of the debt profile. Consumption in period $t = 1$ is

$$c_1^i(z, \phi_1^i, \varphi_1^i) = \phi_1^i e_1(z) + B_0 \left(1 - \eta_1^i\right) + A_0 d_1(z) \left(1 - \eta_1^i\right) - \eta_1^i g_1(z)$$

The price of the risk-free asset is given by the standard formula

$$p_0 = \mathbb{E}\left[\beta \frac{u'(c_1^i(z, \phi_1^i, \varphi_1^i))}{u'(c_0^i)}\right]$$

Differentiating with respect to B_0 and since tax shocks are entirely unanticipated $\mathbb{E}[\eta_1^i] = 1$ gives

$$\frac{\partial p_0}{\partial B_0} = -\frac{\beta}{u'(c_0^i)} \mathbb{C}\text{ov}\left[u''(c_1^i(z, \phi_1^i, \varphi_1^i)), \eta_1^i\right]$$

The same argument applies to find the expression for $\frac{\partial p_0}{\partial A_0}$. ■

A.1.3 Proof of Corollary 1

Proof. From Proposition 1, $\frac{\partial p_0}{\partial B_0} = 0$is zero if (i) u'' is constant or (ii) if $\eta_1^i = 1$. ■

A.1.4 Proof of Proposition 2

Proof. Let $\lambda_0 \geqslant 0$ and $\lambda(z, \phi_1^i, \varphi_1^i) \geqslant 0$ denote the Lagrange multiplier on to the individual budget constraints. The Lagrangian associated to each households' problem is written as

$$\begin{aligned}\mathcal{L}(c_t^i, a_0^i, b_0^i, \lambda_t^i) = u\left(c_0^i\right) + \beta \sum_z \sum_y \pi(z, \phi_1^i, \varphi_1^i) u\left(c_1^i\right) + \lambda_0 \left[e_0 - T_0^i - c_0^i - b_0^i p_0 - a_0^i q_0\right] \\ - \sum_z \sum_y \lambda(z, \phi_1^i, \varphi_1^i) \left[\phi_1^i e_1(z) - T_1^i(z, \phi_1^i, \varphi_1^i) + b_0^i + a_0^i d_1(z) - c_1^i\right]\end{aligned}$$

The first-order conditions for consumption and asset holdings are

$$u'\left(c_0^i\right) = \lambda_0$$
$$\beta\pi(z,y)u'\left(c_1^i(z,\phi_1^i,\varphi_1^i)\right) = \lambda(z,\phi_1^i,\varphi_1^i)$$

and

$$p_0 = \frac{\sum_z \sum_y \lambda(z,\phi_1^i,\varphi_1^i)}{\lambda_0}$$
$$q_0 = \frac{\sum_z \sum_y \lambda(z,\phi_1^i,\varphi_1^i)d_1(z)}{\lambda_0}$$

Hence, it follows that $p_0 = \mathbb{E}_0[m_1^i]$ and $q_0 = \mathbb{E}_0[m_1^i d_1(z)]$ where

$$m_1^i(z,\phi_1^i,\varphi_1^i) = \beta\frac{u'(c_1^i(z,\phi_1^i,\varphi_1^i))}{u'(c_0)}$$

Then, multiplying agent i's budget constraint by $m_1^i(z,\phi_1^i,\varphi_1^i)$ and summing across states gives

$$c_0^i + \mathbb{E}_0[m_1^i c_1^i] = e_0^i + \mathbb{E}_0[m_1^i \phi_1^i e_1^i] - T_0^i - \mathbb{E}_0[m_1^i T_1^i]$$

Using the government budget constraint to substitute out taxes yields

$$\begin{aligned} T_0^i + \mathbb{E}_0[m_1^i \eta_1^i T_1^i(z)] &= g_0 + \mathbb{E}_0[m_1^i \eta_1^i g_1] + B_0\left(\mathbb{E}_0[\eta_1^i m_1^i] - p_0\right) + A_0\left(\mathbb{E}_0[\eta_1^i m_1^i d_1] - q_0\right) \\ &= g_0 + \mathbb{E}_0[m_1^i \eta_1^i g_1] + B_0\mathbb{E}_0[(\eta_1^i - 1)m_1^i] + A_0\mathbb{E}_0[(\eta_1^i - 1)m_1^i d_1] \end{aligned}$$

so that

$$c_0^i + \mathbb{E}_0[m_1^i c_1^i] = e_0^i + \mathbb{E}_0[m_1^i \phi_1^i e_1^i] - g_0 - \mathbb{E}_0[m_1^i \eta_1^i g_1(z)] + \Delta(\mathcal{B})$$

where $\mathbb{E}_0$ is taken with respect to the joint distribution $\pi(z,y)$ and

$$\Delta(\mathcal{B}) \doteq B_0\mathbb{E}_0\left[(1-\eta_1^i)m_1^i\right] + A_0\mathbb{E}_0\left[(1-\eta_1^i)m_1^i d_1(z)\right]$$

is the present value contribution of interest repayments to households' wealth. Because all agents are ex-ante identical, the relative tax burden must be a martingale, and $\mathbb{E}[\eta_1^i] = 1$. Thus

$$\begin{aligned} \Delta(\mathcal{B}) &\doteq B_0\mathbb{E}_0\left[(1-\eta_1^i)m_1^i\right] + A_0\mathbb{E}_0\left[(1-\eta_1^i)m_1^i d_1(z)\right] \\ &= -B_0\mathbb{C}\text{ov}_0(\eta_1^i, m_1^i) - A_0\mathbb{E}_0[d_1\mathbb{C}\text{ov}_0(\eta_1^i, m_1^i|z)] \end{aligned}$$

where the second line follows from using $\mathbb{E}_0[\eta_1^i m_1^i] = \mathbb{E}_0[\eta_1^i]E_0[m_1^i] + \mathbb{C}\text{ov}_0(\eta_1^i, m_1^i)$ and applying the Law of Iterated Expectations. The term $\mathbb{C}\text{ov}_0(\eta_1^i, m_1^i|z)$ is the covariance between marginal utility and the relative tax burden conditional on the aggregate state z. In deriving the second line, it is convenient to define the projection of all individual marginal utility onto the space of tradable payoffs $m^*(z) = \mathbb{E}_0[m(z,y)|z]$. Standard arguments imply that $m^*(z)$ is unique in the asset span. It then follows that $\mathbb{E}[d_1 \mathbb{E}[m^i|z]\mathbb{E}[\eta_1^i|z]] = 1\mathbb{E}[d_1 m^*] = 0$.

The last step is to show that $\mathbb{C}\text{ov}_0(\eta_1^i, m_1^i|z) = 0$ for all z implies $\mathbb{C}\text{ov}_0(\eta_1^i, m_1^i) = 0$. This follows from the Law of Total Covariance and the fact that $\mathbb{E}[\eta_1^i|z] = 1$ is a constant that does not depend on z. If $A_0 = 0$ and $B_0 > 0$, then $\mathbb{C}\text{ov}_0(\eta_1^i, m_1^i) = 0$ implies $\Delta(\mathcal{B}) = 0$. If the government also issues state-contingent debt, then $\mathbb{C}\text{ov}_0(\eta_1^i, m_1^i|z) = 0$ is sufficient for $\Delta(\mathcal{B}) = 0$. ■

A.1.5 Proof of Proposition 3

Proof. Ex-ante symmetry implies that $c_0^i = e_0 - g_0$ is independent of fiscal policy. Further, since all agents choose the same portfolio, I can write $a_0^i = a_0 = 1A_0$ and $b_0^i = b_0 = 1B_0$. Hence

$$\begin{aligned} c_1^i(z, \phi_1^i, \varphi_1^i) &= \phi_1^i(z, y)e_1(z) + 1B_0 + 1A_0 d_1(z) - \eta_1^i[g_1(z) + B_0 + A_0 d_1(z)] \\ &= \phi_1^i(z, y)e_1(z) - \eta_1^i g_1(z) + [1 - \eta_1^i]B_0 + [1 - \eta_1^i]A_0 d_1(z) \end{aligned}$$

Hence, the first-order conditions with respect to B_0 and A_0 are

$$\begin{aligned} \beta \sum_z \sum_y \pi(z, \phi_1^i, \varphi_1^i) u'(c_1^i(z, \phi_1^i, \varphi_1^i)) \left[1 - \eta_1^i\right] &= 0 \qquad : \qquad B_0 \\ \beta \sum_z \sum_y \pi(z, \phi_1^i, \varphi_1^i) u'(c_1^i(z, \phi_1^i, \varphi_1^i)) \left[1 - \eta_1^i\right] d_1(z) &= 0 \qquad : \qquad A_0 \end{aligned}$$

Dividing both sides by $u'(c_0^i) > 0$ and defining

$$m_1^i(z, \phi_1^i, \varphi_1^i) = \beta \frac{u'(c_1^i(z, \phi_1^i, \varphi_1^i))}{u'(c_0^i)}$$

gives equations (12a) and (12b). Further, I can write the first-order conditions as

$$\begin{aligned} \mathbb{E}_0 \left[m_1^i(1 - \eta_1^i)\right] &= 0 \\ \mathbb{E}_0 \left[m_1^i(1 - \eta_1^i)d_1\right] &= \mathbb{E}_0 \left[\mathbb{E}_0[m_1^i(1 - \eta_1^i)d_1|z]\right] = \mathbb{E}_0 \left[d_1 \mathbb{E}_0 \left[m_1^i(1 - \eta_1^i)|z\right]\right] = 0 \end{aligned}$$

where I use the Law of Iterated Expectations. The expectation is with respect to the joint distribution $\pi(z, y^i)$. The first line implies

$$\mathbb{E}_0 \left[m_1^i(1 - \eta_1^i)\right] = p_0(1 - \mathbb{E}[m_1^i \eta_1^i]) = -\mathbb{C}\text{ov}(m_1^i, \eta_1^i) = 0$$

A similar argument gives

$$\mathbb{E}_0 \left[d_1 \mathbb{E}_0 \left[m_1^i(1 - \eta_1^i)|z\right]\right] = -\mathbb{E}_0 \left[d_1 \mathbb{C}\text{ov}(m_1^i, \eta_1^i|z)\right] = 0$$

Sufficient conditions are $\mathbb{C}\text{ov}(m_1^i, \eta_1^i|z) = 0$ and $\mathbb{C}\text{ov}(m_1^i, \eta_1^i) = 0$. Hence, at the optimum, it holds that $\Delta(\mathcal{B}^*) = 0$. Finally, the no-trade equilibrium $B_0 = A_0 = 0$ is not consistent with these conditions. This is because the tax burden and idiosyncratic shocks can be correlated. To see why, note that if $\kappa = 0$, then $\eta_1^i = \phi_1^i$. This is the desired result. ■

A.1.6 Proof of Proposition 4

Proof. With foreign investors, the market clearing condition for risk-free debt is

$$\int_{\mathcal{I}} b_0^i di + F(p_0) = B_0$$

Given that all domestic households choose the same portfolio, it immediately follows that

$$b_0^i = B_0 - F(p_0)$$

Plugging this back into the individual budget constraint and solving for consumption gives

$$\begin{aligned} c_0^i &= e_0 - g_0 \\ c_1^i &= \phi_1^i e_1(z) + A_0 d_1(z)\left(1-\eta_1^i\right) - \eta_1^i g_1(z) - F(p_0) - (1-\eta_1^i)B_0 \end{aligned}$$

where

$$F(p_0) = \alpha_F - \beta_F p_0$$

In the special case that foreign demand is inelastic ($\beta_F = 0$)

$$\frac{\partial c_1^i}{\partial \alpha_F} = -1$$

so that period $t = 1$ consumption decline with the size of the foreign sector. ■

A.1.7 Proof of Lemma 2

Proof. Standard arguments imply that the ratio of marginal utilities is a valid SDF. The Euler equations are

$$\begin{aligned} p_t &= \mathbb{E}_t[m_{t+1}^i] \\ q_t &= \mathbb{E}_t[m_{t+1}^i d_{t+1}] \end{aligned}$$

Multiplying the $t+1$ budget constraint by m_{t+1}^i and taking expectation gives

$$\begin{aligned} &\mathbb{E}_t\left[m_{t+1}^i c_{t+1}^i\right] + \mathbb{E}_t\left[m_{t+1}^i p_{t+1}^i b_{t+1}^i\right] + \mathbb{E}_t\left[m_{t+1}^i q_{t+1}^i a_{t+1}^i\right] \\ &\qquad = \mathbb{E}_t\left[m_{t+1}^i b_t^i\right] + \mathbb{E}_t\left[m_{t+1}^i d_{t+1} a_t^i\right] + \mathbb{E}_t\left[m_{t+1}^i \phi_{t+1}^i e_{t+1}\right] - \mathbb{E}_t\left[m_{t+1}^i T_{t+1}^i\right] \end{aligned}$$

or

$$\begin{aligned} &\mathbb{E}_t\left[m_{t+1}^i c_{t+1}^i\right] + \mathbb{E}_t\left[m_{t+1}^i p_{t+1}^i b_{t+1}^i\right] + \mathbb{E}_t\left[m_{t+1}^i q_{t+1}^i a_{t+1}^i\right] \\ &\qquad = p_t^i b_t^i + q_t^i a_t^i + \mathbb{E}_t\left[m_{t+1}^i \phi_{t+1}^i e_{t+1}\right] - \mathbb{E}_t\left[m_{t+1}^i T_{t+1}^i\right] \end{aligned}$$

Substituting back into the time t budget constraint gives

$$c_t^i + \mathbb{E}_t\left[m_{t+1}^i c_{t+1}^i\right] + \mathbb{E}_t\left[m_{t+1}^i\left(p_{t+1}^i b_{t+1}^i + q_{t+1}^i a_{t+1}^i\right)\right]$$
$$= b_{t-1}^i + d_t a_{t-1}^i + \phi_t^i e_t + \mathbb{E}_t\left[m_{t+1}^i \phi_{t+1}^i e_{t+1}\right] - T_t^i - \mathbb{E}_t\left[m_{t+1}^i T_{t+1}^i\right]$$

Then, multiplying the $t+2$ budget constraint by m_{t+2}^i and taking expectations conditional on information at time $t+1$ gives

$$\mathbb{E}_{t+1}\left[m_{t+2}^i c_{t+2}^i\right] + \mathbb{E}_{t+1}\left[m_{t+2}^i p_{t+2}^i b_{t+2}^i\right] + \mathbb{E}_{t+1}\left[m_{t+2}^i q_{t+2}^i a_{t+2}^i\right]$$
$$= \mathbb{E}_{t+1}\left[m_{t+2}^i b_{t+1}^i\right] + \mathbb{E}_{t+1}\left[m_{t+2}^i d_{t+2} a_{t+1}^i\right] + \mathbb{E}_{t+1}\left[m_{t+2}^i \phi_{t+2}^i e_{t+2}\right] - \mathbb{E}_{t+1}\left[m_{t+2}^i T_{t+2}^i\right]$$

or

$$\mathbb{E}_{t+1}\left[m_{t+2}^i c_{t+2}^i\right] + \mathbb{E}_{t+1}\left[m_{t+2}^i p_{t+2}^i b_{t+2}^i\right] + \mathbb{E}_{t+1}\left[m_{t+2}^i q_{t+2}^i a_{t+2}^i\right]$$
$$= p_{t+1}^i b_{t+1}^i + a_{t+1}^i q_{t+1}^i + \mathbb{E}_{t+1}\left[m_{t+2}^i d_{t+2} a_{t+1}^i\right] + \mathbb{E}_{t+1}\left[m_{t+2}^i \phi_{t+2}^i e_{t+2}\right] - \mathbb{E}_{t+1}\left[m_{t+2}^i T_{t+2}^i\right]$$

Plugging this back into the time t budget constraint and using the Law of Iterated Expectations yields

$$c_t^i + \mathbb{E}_t\left[m_{t+1}^i c_{t+1}^i\right] + \mathbb{E}_t\left[m_{t|t+2}^i c_{t+2}^i\right] + \mathbb{E}_t\left[m_{t|t+2}^i\left(p_{t+2}^i b_{t+2}^i + q_{t+2}^i a_{t+2}^i\right)\right]$$
$$= b_{t-1}^i + d_t a_{t-1}^i + \phi_t^i e_t + \mathbb{E}_t\left[m_{t+1}^i \phi_{t+1}^i e_{t+1}\right] + \mathbb{E}_t\left[m_{t|t+2}^i \phi_{t+2}^i e_{t+2}\right]$$
$$- T_t^i - \mathbb{E}_t\left[m_{t+1}^i T_{t+1}^i\right] - \mathbb{E}_t\left[m_{t|t+2}^i T_{t+1}^i\right]$$

where $m_{t|t+2}^i = m_{t+1}^i m_{t+2}^i$ is the product of adjacent one-period stochastic discount factors. Iterating forward until time T gives

$$\sum_{j=0}^{T} \mathbb{E}_t[m_{t|t+j}^i c_{t+j}^i] + \mathbb{E}_t\left[m_{t|t+T}^i(p_{t+T} b_{t+T}^i + q_{t+T} a_{t+T}^i)\right]$$
$$= b_{t-1}^i + d_t a_{t-1}^i + \sum_{j=0}^{T} \mathbb{E}_t[m_{t|t+j}^i \phi_{t+j}^i e_{t+j}] - \sum_{j=0}^{T} \mathbb{E}_t[m_{t|t+j}^i T_{t+j}^i]$$

The first term on the right-hand side is the financial wealth of household i at the beginning of period t. The second term is the present value of capital income, discounted using household i's marginal utility. The last term is the present value of taxes, discounted again using household i's marginal utility.

Let $\eta_t^i = \frac{T_t^i}{T_t}$ denote the relative tax-burden across agents at time t. The present value relation above becomes

$$\sum_{j=0}^{T} \mathbb{E}_t[m^i_{t|t+j} c^i_{t+j}] + \mathbb{E}_t \left[m^i_{t|t+T} (p_{t+T} b^i_{t+T} + q_{t+T} a^i_{t+T}) \right]$$
$$= b^i_{t-1} + d_t a^i_{t-1} + \sum_{j=0}^{T} \mathbb{E}_t[m^i_{t|t+j} \phi^i_{t+j} e_{t+j}] - \sum_{j=0}^{T} \mathbb{E}_t[m^i_{t|t+j} \eta^i_{t+j} T_{t+j}]$$

I use the government budget constraint to substitute taxes, given that

$$g_t + B_{t-1} + A_{t-1} d_t = p_t B_t + q_t A_t + T_t$$

As a result

$$\begin{aligned}
\sum_{j=0}^{T} \mathbb{E}_t[m^i_{t|t+j} \eta^i_{t+j} T_{t+j}] &= \sum_{j=0}^{T} \mathbb{E}_t \left[m^i_{t|t+j} \eta^i_{t+j} \left(g_{t+j} + B_{t-j-1} + A_{t+j-1} d_t - p_{t+j} B_{t+j} - q_{t+j} A_{t+j} \right) \right] \\
&= \sum_{j=0}^{T} \mathbb{E}_t \left[m^i_{t|t+j} \eta^i_{t+j} g_{t+j} \right] + \eta^i_t B_{t-1} + \eta^i_t A_{t-1} d_t \\
&\quad + \sum_{j=0}^{T} \mathbb{E}_t \left[B_{t+j} \left(m^i_{t|t+j+1} \eta^i_{t+j+1} - p_{t+j} m^i_{t|t+j} \eta^i_{t+j} \right) \right] \\
&\quad + \sum_{j=0}^{T} \mathbb{E}_t \left[A_{t+j} \left(m^i_{t|t+j+1} \eta^i_{t+j+1} d_{t+j+1} - q_{t+j} m^i_{t|t+j} \eta^i_{t+j} \right) \right] \\
&\quad - \mathbb{E}_t \left[m^i_{t|t+T} \eta^i_{t+T} \left(p_{t+T} B_{t+T} + q_{t+T} A_{t+T} \right) \right] \\
&= \sum_{j=0}^{T} \mathbb{E}_t \left[m^i_{t|t+j} \eta^i_{t+j} g_{t+j} \right] - \Delta(\boldsymbol{\mathcal{B}})^T_t
\end{aligned}$$

where the contribution of interest spending is

$$\begin{aligned}
\Delta(\boldsymbol{\mathcal{B}})^T_t = \mathbb{E}_t \left[m^i_{t|t+T} \eta^i_{t+T} \left(p_{t+T} B_{t+T} + q_{t+T} A_{t+T} \right) \right] - \eta^i_t B_{t-1} - \eta^i_t A_{t-1} d_t \\
+ \sum_{j=0}^{T} \mathbb{E}_t \left[B_{t+j} \left(p_{t+j} m^i_{t|t+j} \eta^i_{t+j} - m^i_{t|t+j+1} \eta^i_{t+j+1} \right) \right] \\
+ \sum_{j=0}^{T} \mathbb{E}_t \left[A_{t+j} \left(q_{t+j} m^i_{t|t+j} \eta^i_{t+j} - m^i_{t|t+j+1} \eta^i_{t+j+1} d_{t+j+1} \right) \right]
\end{aligned}$$

This term is the infinite horizon counterpart of $\Delta(\mathcal{B})$ and gives the desired result. Let $\Delta(\widetilde{\boldsymbol{\mathcal{B}}})^T_t$ denote the present value of interest expense if the government only issues risk-free debt

$$\Delta(\widetilde{\boldsymbol{\mathcal{B}}})^T_t = \mathbb{E}_t \left[m^i_{t|t+T} \eta^i_{t+T} \left(p_{t+T} B_{t+T} \right) \right] - \eta^i_t B_{t-1} + \sum_{j=0}^{T} \mathbb{E}_t \left[B_{t+j} m^i_{t|t+j} \left(p_{t+j} \eta^i_{t+j} - m^i_{t+j+1} \eta^i_{t+j+1} \right) \right]$$

which concludes the derivation. ■

A.1.8 Proof of Proposition 6

Proof. Part (c) follows from direct application of the Law of Total Covariance. Let x_{t+T} denote a given cash flow that only depends on the aggregate history up to z_{t+T}. Then

$$\begin{aligned}
\mathbb{E}_t\left[m^i_{t|t+T}\eta^i_{t+T}x_{t+T}\right] &= \mathbb{E}_t\left[m^i_{t|t+T}x_{t+T}\right]\mathbb{E}_t\left[\eta^i_{t+T}\right] + \mathbb{C}\mathrm{ov}_t(m^i_{t|t+T}x_{t+T}, \eta^i_{t+T}) \\
&= \mathbb{E}_t\left[m^i_{t|t+T}x_{t+T}\right]\mathbb{E}_t\left[\eta^i_{t+T}\right] + \mathbb{E}_t\left[x_{t+T}\mathbb{C}\mathrm{ov}\left(m^i_{t|t+T}, \eta^i_{t+T}|z_{t+T}\right)\right] \\
&\qquad + \mathbb{C}\mathrm{ov}_t\left(\mathbb{E}[m^i_{t|t+T}x_{t+T}|z_{t+T}], \mathbb{E}[\eta^i_{t+T}|z_{t+T}]\right) \\
&= \mathbb{E}_t\left[m^i_{t|t+T}x_{t+T}\right]\mathbb{E}_t\left[\eta^i_{t+T}\right] + \mathbb{E}_t\left[x_{t+T}\mathbb{C}\mathrm{ov}\left(m^i_{t|t+T}, \eta^i_{t+T}|z_{t+T}\right)\right] \\
&\leqslant \mathbb{E}_t\left[m^i_{t|t+T}x_{t+T}\right]\mathbb{E}_t\left[\eta^i_{t+T}\right]
\end{aligned}$$

where the inequality follows from the premise that $\mathbb{C}\mathrm{ov}_t(\eta^i_{t+1}, m^i_{t+1}) < 0$. Hence

$$\lim_{T\to\infty}\mathbb{E}_t\left[m^i_{t|t+T}\eta^i_{t+T}x_{t+T}\right] \leqslant \lim_{T\to\infty}\mathbb{E}_t\left[m^i_{t|t+T}x_{t+T}\right]\mathbb{E}_t\left[\eta^i_{t+T}\right]$$

Replacing $x_{t+T} = p_{t+T}B_{t+T} + q_{t+T}A_{t+T}$ in the last equality, invoking the government transversality condition, and using the fact that $\mathbb{E}_t\left[\eta^i_{t+T}\right]$ is bounded gives

$$\lim_{T\to\infty}\mathbb{E}_t\left[m^i_{t|t+T}\eta^i_{t+T}\left(p_{t+T}B_{t+T} + q_{t+T}A_{t+T}\right)\right] = 0$$

which is (tTVC) and gives the desired result.

For ease of notation, I prove the claim in two steps. I first study a case in which the government only issues risk-free debt. This is not strictly necessary but it helps to gain intuition without complicating notation. I then argue that the same steps go through when government debt issues state-contingent debt.

The household's dynamic budget constraint is

$$c^i_t + p_t b^i_t = b^i_{t-1} + \phi^i_t e_t - T^i_t$$

Multiplying the time $t+1$ budget constraint by m^i_{t+1} and taking expectations yields

$$\mathbb{E}_t\left[m^i_{t+1}c^i_{t+1}\right] + \mathbb{E}_t\left[m^i_{t+1}p_{t+1}b^i_{t+1}\right] = p_t b^i_t + \mathbb{E}_t\left[m^i_{t+1}\phi^i_{t+1}e_{t+1}\right] - \mathbb{E}_t\left[m^i_{t+1}T^i_{t+1}\right]$$

Repeating the same steps with the time $t+2$ budget constraint gives

$$\begin{aligned}
&\mathbb{E}_{t+1}\left[m^i_{t+2}c^i_{t+2}\right] + \mathbb{E}_{t+1}\left[m^i_{t+2}p_{t+2}b^i_{t+2}\right] = b^i_{t+1}p_{t+1} + \mathbb{E}_{t+1}\left[m^i_{t+2}\phi^i_{t+2}e_{t+2}\right] - \mathbb{E}_{t+1}\left[m^i_{t+2}T^i_{t+2}\right] \\
&\implies b^i_{t+1}p_{t+1} = \mathbb{E}_{t+1}\left[m^i_{t+2}c^i_{t+2}\right] + \mathbb{E}_{t+1}\left[m^i_{t+2}p_{t+2}b^i_{t+2}\right] - \mathbb{E}_{t+1}\left[m^i_{t+2}\phi^i_{t+2}e_{t+2}\right] + \mathbb{E}_{t+1}\left[m^i_{t+2}T^i_{t+2}\right]
\end{aligned}$$

Substituting back into the time t budget constraint and solving forward until time $t+T$ gives

$$\mathbb{E}_t\left[\sum_{j=0}^{T} m^i_{t|t+j} c^i_{t+j}\right] + \mathbb{E}_t\left[m^i_{t|t+T} b^i_{t+T} p_{t+T}\right] = \mathbb{E}_t\left[\sum_{j=0}^{T} m^i_{t|t+j} \phi^i_{t+j} e_t\right] - \mathbb{E}_t\left[\sum_{j=0}^{T} m^i_{t|t+j} T^i_{t+j}\right]$$

I use the government budget constraint to separate the present value of taxes that finance government purchases from the present value of taxes to finance interest expense. The government budget constraint is

$$g_t + p_t B_t = B_{t-1} + T_t$$

Solving for T_t and substituting into the household's present value constraint gives

$$\begin{aligned}\mathbb{E}_t\left[\sum_{j=0}^{T} m^i_{t|t+j} T^i_{t+j}\right] &= \mathbb{E}_t\left[\sum_{j=0}^{T} m^i_{t|t+j} \eta^i_{t+j} g_{t+j}\right] + \mathbb{E}_t\left[\sum_{j=0}^{T} m^i_{t|t+j} \eta^i_{t+j} \left(p_{t+j} B_{t+j} - B_{t+j-1}\right)\right] \\ &= \mathbb{E}_t\left[\sum_{j=0}^{T} m^i_{t|t+j} \eta^i_{t+j} g_{t+j}\right] - \eta^i_t B_{t-1} + \mathbb{E}_t\left[m^i_{t|t+T} \eta^i_{t+T} p_{t+T} B_{t+T}\right] \\ &\quad + \sum_{j=1}^{T} \mathbb{E}_t\left[B_{t+j} m^i_{t|t+j}\left(\eta^i_{t+j} p_{t+j} - m^i_{t+j+1} \eta^i_{t+j+1}\right)\right]\end{aligned}$$

With a slight abuse of notation, I define $\Delta(\boldsymbol{\mathcal{B}})^T_t$ as

$$\Delta(\boldsymbol{\mathcal{B}})^T_t \doteq \sum_{j=1}^{T} \mathbb{E}_t\left[B_{t+j} m^i_{t|t+j}\left(\eta^i_{t+j} p_{t+j} - m^i_{t+j+1} \eta^i_{t+j+1}\right)\right] - \eta^i_t B_{t-1} + \mathbb{E}_t\left[m^i_{t|t+T} \eta^i_{t+T} p_{t+T} B_{t+T}\right]$$

This is present value contribution of interest expense. The households' budget constraint thus becomes

$$\mathbb{E}_t\left[\sum_{j=0}^{T} m^i_{t|t+j} c^i_{t+j}\right] + \mathbb{E}_t\left[m^i_{t|t+T} b^i_{t+T} p_{t+T}\right] = \mathbb{E}_t\left[\sum_{j=0}^{T} m^i_{t|t+j} \phi^i_{t+j} e_t\right] - \mathbb{E}_t\left[\sum_{j=0}^{T} m^i_{t|t+j} \eta^i_{t+j} g_{t+j}\right] + \Delta(\boldsymbol{\mathcal{B}})^T_t$$

It is standard to require for individual optimality that

$$\lim_{T\to\infty} \mathbb{E}_t\left[m^i_{t|t+T} b^i_{t+T} p_{t+T}\right] = 0$$

However, tax liabilities are not tradable, so that household's optimality is silent about $\Delta(\boldsymbol{\mathcal{B}})^T_t$.

Case 1: I show that when the relative tax burden is (i) a martingale and (ii) it is uncorrelated to marginal utility, the first summation in $\Delta(\boldsymbol{\mathcal{B}})^T_t$ is equal to zero.

Suppose (i) η^i_{t+j} is a martingale and (ii) $\mathbb{C}\text{ov}_{t+j}(\eta^i_{t+j+1}, m^i_{t+j+1}) = 0$. Each term in the summation becomes

$$
\begin{aligned}
\mathbb{E}_t \left[B_{t+j} m^i_{t|t+j} \left(\eta^i_{t+j} p_{t+j} - m^i_{t+j+1} \eta^i_{t+j+1} \right) \right] &\overset{\text{LIE}}{=} \mathbb{E}_t \left[B_{t+j} m^i_{t|t+j} \left(\eta^i_{t+j} p_{t+j} - \mathbb{E}_{t+j} \left[m^i_{t+j+1} \eta^i_{t+j+1} \right] \right) \right] \\
&\overset{\text{(ii)}}{=} \mathbb{E}_t \left[B_{t+j} m^i_{t|t+j} \left(\eta^i_{t+j} p_{t+j} - \mathbb{E}_{t+j} \left[m^i_{t+j+1} \right] \mathbb{E}_{t+j} \left[\eta^i_{t+j+1} \right] \right) \right] \\
&\overset{\text{(i)}}{=} \mathbb{E}_t \left[B_{t+j} m^i_{t|t+j} \left(\eta^i_{t+j} p_{t+j} - p_{t+j} \eta^i_{t+j} \right) \right] = 0
\end{aligned}
$$

Hence, under (i) and (ii) the present value of interest expense $\Delta(\boldsymbol{\mathcal{B}})^T_t$ collapses to

$$
\Delta(\boldsymbol{\mathcal{B}})^T_t = \mathbb{E}_t \left[m^i_{t|t+T} \eta^i_{t+T} p_{t+T} B_{t+T} \right] - \eta^i_t B_{t-1} = \eta^i_t \left(\mathbb{E}_t \left[m^i_{t|t+T} p_{t+T} B_{t+T} \right] - B_{t-1} \right)
$$

The transversality condition alone is not sufficient to conclude that

$$
\lim_{T \to \infty} \mathbb{E}_t \left[m^i_{t|t+T} \eta^i_{t+T} p_{t+T} B_{t+T} \right] = 0
$$

Case 2: I next study the case in which the relative tax burden is (i) a martingale, but (ii) The tax burden is negatively correlated with marginal utility such that $\mathbb{C}\text{ov}_{t+j}(\eta^i_{t+j+1}, m^i_{t+j+1}) < 0$. Each term now becomes

$$
\begin{aligned}
\mathbb{E}_t \left[B_{t+j} m^i_{t|t+j} \left(\eta^i_{t+j} p_{t+j} - m^i_{t+j+1} \eta^i_{t+j+1} \right) \right] &\overset{\text{LIE}}{=} \mathbb{E}_t \left[B_{t+j} m^i_{t|t+j} \left(\eta^i_{t+j} p_{t+j} - \mathbb{E}_{t+j} \left[m^i_{t+j+1} \eta^i_{t+j+1} \right] \right) \right] \\
&\overset{\text{(ii)}}{=} \mathbb{E}_t \left[B_{t+j} m^i_{t|t+j} \left(\eta^i_{t+j} p_{t+j} - \mathbb{E}_{t+j} \left[m^i_{t+j+1} \right] \mathbb{E}_{t+j} \left[\eta^i_{t+j+1} \right] - \mathbb{C}\text{ov}_{t+j}(\eta^i_{t+j+1}, m^i_{t+j+1}) \right) \right] \\
&\overset{\text{(i)}}{=} -\mathbb{E}_t \left[B_{t+j} m^i_{t|t+j} \mathbb{C}\text{ov}_{t+j}(\eta^i_{t+j+1}, m^i_{t+j+1}) \right] > 0
\end{aligned}
$$

The inequality uses $B_{t+j} > 0$ and $m^i_{t|t+j} > 0$, so $\Delta(\boldsymbol{\mathcal{B}})^T_t > 0$. Debt issuance improves risk sharing. ■

A.2 Proofs for Section 5

A.2.1 Proof of Lemma 3

Proof. As in the main text, define the after-tax net wealth w^i_t as

$$
w^i_t = n^i_t - T^i_t
$$

Financial assets evolve as

$$
dn^i_t = b^i_t r_t dt + q_t k^i_t \mathbb{E}[dR^i_t(\iota)] - c^i_t dt - \tau^i_t dt + q_t k^i_t \sigma^R_t dZ^i_t
$$

and let

$$
dT^i_t = \mu^T_t T^i_t dt + \sigma^T_t T^i_t dZ^i_t - \tau^i_t dt
$$

Conjecture

$$
V(n_t, T_t) = \frac{1}{\rho} \log(n_t - T_t) + \zeta
$$

where ζ is a constant. The derivatives are

$$V_n = \frac{1}{\rho w_t} \quad : \quad V_{nn} = -\frac{1}{\rho w_t^2}$$
$$V_T = -\frac{1}{\rho w_t} \quad : \quad V_{TT} = \frac{1}{\rho w_t^2}$$
$$V_{nT} = \frac{1}{\rho w_t^2}$$

Thus, because the τ_t term drops (offsetting terms in dT_t^i and dn_t^i), I get

$$\mathcal{D}V_t = \frac{1}{\rho w_t}\left[b_t r_t + q_t k_t \mathbb{E}[dR_t^i(\iota)] - c_t\right] - \frac{1}{2}\frac{1}{\rho w_t^2}\sigma_t^2 (q_t k_t)^2 - \frac{1}{\rho w_t}\mu_t^T T_t + \frac{1}{2}\frac{1}{\rho w_t^2}(\sigma_t^T)^2 T_t^2 + \frac{1}{\rho w_t^2}\sigma_t^T \sigma_t T_t q_t k_t$$

so that the HJB is

$$\max_{c_t,\iota,k_t,b_t}\left\{\frac{1}{\rho w_t}\left[b_t r_t + q_t k_t \mathbb{E}[dR_t^i(\iota)] - c_t\right] - \frac{1}{2}\frac{1}{\rho w_t^2}\sigma_t^2 (q_t k_t)^2 - \frac{1}{\rho w_t}\mu_t^T T_t + \frac{1}{2}\frac{1}{\rho w_t^2}(\sigma_t^T)^2 T_t^2 - \frac{1}{\rho w_t^2}\sigma_t^T \sigma_t T_t q_t k_t - \log w_t^i - \rho\zeta + \log c_t^i\right\} = 0$$

The equation is rewritten as

$$\max_{c_t,\iota,k_t,b_t}\left\{\varphi_t^b r_t + \varphi_t^k \mathbb{E}[dR_t^i(\iota)] - \frac{c_t}{w_t} - \frac{1}{2}\sigma_t^2(\varphi_t^k)^2 - \mu_t^T \varphi_t^T + \frac{1}{2}(\sigma_t^T)^2(\varphi_t^T)^2 + \sigma_t^T \sigma_t \varphi_t^T \varphi_t^k - \rho \log w_t^i - \rho^2 \zeta_t + \rho \log c_t^i\right\} = 0$$

where

$$\varphi_t^k = \frac{q_t k_t}{w_t} \quad : \quad \varphi_t^b = \frac{b_t^i}{w_t} \quad : \quad \varphi_t^T = \frac{T_t^i}{w_t^i}$$

and $\varphi_t^k + \varphi_t^b - \varphi_t^T = 1$. Note that

$$\varphi_t^k + \varphi_t^b = 1 + \varphi_t^T \Longrightarrow \frac{\varphi_t^k}{1+\varphi_t^T} + \frac{\varphi_t^b}{1+\varphi_t^T} = 1$$

Define $\theta_t^k = \frac{\varphi_t^k}{1+\varphi_t^T}$ and $\theta_t^b = (1-\theta_t^k)$. Then

$$\max_{c_t,\iota,\theta_t^k}\left\{(1+\varphi_t^T)\left[r_t + \theta_t^k(\mathbb{E}[dR_t^i(\iota)] - r_t)\right] - \frac{c_t}{w_t} - \frac{1}{2}\sigma_t^2(1+\varphi_t^T)^2(\theta_t^k)^2 - \mu_t^T \varphi_t^T dt + \frac{1}{2}(\sigma_t^T)^2(\varphi_t^T)^2 + \sigma_t^T \sigma_t \varphi_t^T (1+\varphi_t^T)\theta_t^k - \rho \log w_t^i - \rho^2 \zeta_t + \rho \log c_t^i\right\} = 0$$

Differentiating with respect to c_t and ι_t, I obtain the first-order conditions

$$c_t^i = \rho w_t^i$$
$$\Phi'(\iota) = \frac{1}{q_t}$$

Then, the first-order condition for portfolio weight θ_t^k is

$$(1 + \varphi_t^T)(\mathbb{E}[dR_t^i(\iota)] - r_t) - \sigma_t^2(1 + \varphi_t^T)^2\theta_t^k + \sigma_t^T\sigma_t\varphi_t^T(1 + \varphi_t^T) = 0$$

or, simplifying

$$\begin{aligned}\mathbb{E}[dR_t^i(\iota)] - r_t &= \sigma_t^2(1 + \varphi_t^T)\theta_t^k - \sigma_t^T\sigma_t\varphi_t^T \\ &= \sigma_t^2\varphi_t^k - \sigma_t\sigma_t^T\varphi_t^T\end{aligned}$$

Everyone chooses the same consumption, investment, and portfolio shares. Market clearing in the capital market gives

$$\varphi_t^k \int_{\mathcal{I}} w_t^i di = qk_t^i$$

hence

$$\varphi_t^k = \frac{qk_t^i}{\int_{\mathcal{I}} w_t^i di}$$

■

A.2.2 Proof of Proposition 7

Proof. I derive equations for the aggregate quantities ι, q, π, and r. I look for an equilibrium in which q_t is constant. Households consume

$$c_t^i = \rho w_t^i$$

Further, because q_t is the same for everyone, ι_t^i is the same for everyone. The aggregate resource constraint is

$$\int_{\mathcal{I}} \rho w_t^i + \iota K_t = aK_t$$

I use that

$$q_t^k k_t^i = \varphi_t^k w_t^i \Longrightarrow w_t^i = \frac{q_t^k k_t^i}{\varphi_t^k}$$

for everyone. This gives

$$\rho\frac{q_t^k K_t}{\varphi_t^k} + \iota K_t = aK_t$$

or

$$\rho\frac{q_t^k}{\varphi_t^k} + \iota_t = a$$

The first-order condition for investment gives

$$q_t^k = 1 + \phi\iota_t$$

Hence

$$\rho\frac{1 + \varrho\iota_t}{\varphi_t^k} + \iota_t = a$$

Solving for ι_t gives

$$\iota_t(\varphi_t^k) = \frac{a\varphi_t^k - \rho}{\rho\varrho + \varphi_t^k}$$

Hence,

$$q_t(\varphi_t^k) = 1 + \varrho\left(\frac{a\varphi_t^k - \rho}{\rho\varrho + \varphi_t^k}\right)$$

It follows that q_t and ι_t can both be written as a function of φ_t^k only. The risk-free rate is then obtain using the first-order condition for portfolio holdings,

$$\mathbb{E}[dR_t^i(\iota)] - r_t = \sigma_t^2\varphi_t^k - \sigma_t\sigma_t^T\varphi_t^T$$

where

$$\mathbb{E}[dR_t^i(\iota)] = \frac{a - \iota}{q_t} + \Phi(\iota) - \delta$$

This gives an expression for the risk-free rate as a function of φ_t^T and φ_t^k.

$$r_t(\varphi_t^k) = \frac{a - \iota_t}{q_t} + \Phi(\iota) - \delta - \nu^2\varphi_t^k + \nu\sigma_t^T\varphi_t^T$$

The missing step is to now characterize the individual SDF ξ_t^i. Consumption is

$$c_t^i = \rho w_t^i$$

and

$$\frac{d\xi_t^i}{\xi_t^i} = -r_t dt - \pi_t dZ_t^i$$

where

$$r_t = \rho + \mu_c - \sigma_c^2$$
$$\pi_t = \sigma_c$$

given the consumption dynamics

$$\frac{dc_t}{c_t} = \mu_c dt + \sigma_c dZ_t^i$$

Using $\rho w_t^i = c_t^i$, using $w_t^i = n_t^i - T_t^i$ and applying ito's lemma on both sides and matching coefficients on the diffusion term immediately yields

$$\sigma_c = \varphi_t^k \sigma_t - \sigma_t^T \varphi_t^T = \pi_t$$

The volatility of the tax claim lowers interest rates through the precautionary savings motive. The asset pricing equation for capital is written as

$$\mathbb{E}[dR_t^i(\iota)] - r_t = \sigma_t \pi_t$$

This is the familiar asset pricing relation that links expected excess return to risk premia. It thus follows that

$$r_t^{**} = r_t + \pi_t \sigma_t^\eta$$

is also written as a function of φ_t^k only. I ■

A.2.3 Proof of Proposition 8

Proof. Recall that each individual households' stochastic discount factor is is

$$\frac{d\xi_t^i}{\xi_t^i} = -r_t dt - \pi_t dZ_t^i$$

Straightforward application of ito's lemma gives

$$\xi_s^i = \xi_t^i \exp\left(-\int_t^s r_u du\right) \exp\left(-\int_t^s \pi_s dZ_s - \frac{1}{2}\int_t^s \pi_s^2 ds\right)$$

and

$$\eta_s^i = \eta_t^i \exp\left(\int_t^s \sigma_u^\eta dZ_u - \frac{1}{2}\int_t^s (\sigma_u^\eta)^2 ds\right)$$

Then, the present discounted value of tax liabilities is given by

$$
\begin{aligned}
T_t^i &= \mathbb{E}_t\left[\int_t^\infty \frac{\xi_s^i}{\xi_t^i}\tau_t^i ds\right] \\
&= \mathbb{E}_t\left[\int_t^\infty \frac{\xi_s^i}{\xi_t^i}\eta_t^i\tau_t ds\right] \\
&= \mathbb{E}_t\left[\int_t^\infty \exp\left(-\int_t^s r_u du - \int_t^s \pi_s dZ_s - \frac{1}{2}\int_t^s \pi_s^2 ds\right)\eta_t^i \exp\left(\int_t^s \sigma_u^\eta dZ_u - \frac{1}{2}\int_t^s (\sigma_u^\eta)^2 ds\right)\tau_t ds\right] \\
&= \eta_t^i \int_t^\infty \exp\left(-\int_t^s r_u du - \frac{1}{2}\int_t^s \pi_s^2 ds - \frac{1}{2}\int_t^s (\sigma_u^\eta)^2 ds\right)\mathbb{E}_t\left[\exp\left(-\int_t^s \pi_s dZ_s + \int_t^s \sigma_u^\eta dZ_u\right)\tau_t ds\right] \\
&= \eta_t^i \int_t^\infty \exp\left(-\int_t^s r_u du - \int_t^s \pi_s \sigma_u^\eta ds\right)\tau_t ds
\end{aligned}
$$

The second line writes individual taxes as $\tau_t^i = \eta_t^i \tau_t$, where τ_t is the aggregate tax revenue. The last equation uses Itô isometry and the fact that the evolution of r_t and π_s is deterministic.

Substituting the government budget constraint

$$(r_t - \mu_t)\mathcal{B}_t = \tau_t$$

and using $d\mathcal{B}_t = \mu_t \mathcal{B}_t$ gives

$$\mathcal{B}_s = \mathcal{B}_t \exp\left(\int_t^s \mu_s ds\right)$$

As a result, the present value of tax liabilities is

$$
\begin{aligned}
T_t^i &= \mathcal{B}_t\eta_t^i \int_t^\infty \exp\left(-\int_t^s r_u du - \int_t^s \pi_s \sigma_u^\eta ds\right)(r_s - \mu_s)\exp\left(\int_t^s \mu_s ds\right) ds \\
&= \mathcal{B}_t\eta_t^i \int_t^\infty \exp\left(-\int_t^s r_u du - \int_t^s \pi_s \sigma_u^\eta ds + \int_t^s \mu_s ds\right)(r_s - \mu_s) ds
\end{aligned}
$$

Aggregating across households and dividing by $W_t = \int_{\mathcal{I}} w_t^i di$ gives

$$\varphi_t^T = \varphi_t^b \int_t^\infty \exp\left(-\int_t^s r_u du - \int_t^s \pi_s \sigma_u^\eta ds + \int_t^s \mu_s ds\right)(r_s - \mu_s) ds$$

which is the desired result. In a BGP where r_t and π_t are constant,

$$
\begin{aligned}
\varphi_t^T &= \varphi_t^b \int_t^\infty \exp\left(-\int_t^s r_u du - \int_t^s \pi_s \sigma_u^\eta ds + \int_t^s \mu_s ds\right)(r_s - \mu_s) ds \\
&= \varphi_t^b \int_t^\infty e^{-(r_s^{**} - \mu_s)(s-t)}(r_s - \mu_s) ds \\
&= -\varphi_t^b \left.\frac{r_s - \mu_s}{r_s^{**} - \mu_s} e^{-(r_s^{**} - \mu_s)(s-t)}\right|_t^\infty \\
&= \varphi_t^b \frac{r_t - \mu_t}{r_t^{**} + \mu_s} \\
&= \varphi_t^b \frac{r_t - \mu_t}{r_t + \pi_t \sigma_t^\eta - \mu_t}
\end{aligned}
$$

This gives the desired result and completes the proof. ■

A.2.4 Proof of Proposition 9

Proof. Differentiating T_t^i with respect to time gives

$$dT_t^i = (r_t T_t^i - \tau_t^i)dt$$

It immediately follows that $\sigma_t^T = 0$. Net wealth w_t^i thus evolves as

$$\begin{aligned} dw_t^i &= dR_t^i(\iota_t)q_t k_t^i + b_t^i r_t dt - (r_t T_t^i - \tau_t^i)dt - c_t^i dt - \tau_t^i dt \\ &= (dR_t^i(\iota_t) - r_t dt)q_t k_t^i + q_t k_t^i r_t dt + b_t^i r_t dt - r_t T_t^i dt - c_t^i dt \\ &= (q_t k_t^i + b_t^i - T_t^i)r_t dt + q_t k_t^i(dR_t^i(\iota_t) - r_t dt) - c_t^i dt \\ &= w_t^i r_t dt + q_t k_t^i(\mathbb{E}[dR_t^i(\iota_t)]dt - r_t dt) - c_t^i dt + q_t k_t^i \nu dZ_t^i \end{aligned}$$

The first-order conditions for consumption and portfolio holdings are

$$\begin{aligned} c_t^i &= \rho w_t^i \\ \mathbb{E}[dR_t(\iota_t)] - r_t &= \varphi_t^k \nu^2 \end{aligned}$$

The share of wealth that is **not** invested in capital is

$$1 - \varphi_t^k = \frac{b_t^i - T_t^i}{q_t k_t^i + b_t^i - T_t^i}$$

Because φ_t^k is the same for everyone, $1 - \varphi_t^k$ must also be common across all households. I thus write

$$(q_t k_t^i + b_t^i - T_t^i)(1 - \varphi_t^k) = b_t^i - T_t^i$$

Aggregating and imposing market clearing in the government bond market $\int_{\mathcal{I}} b_t^i di = \mathcal{B}_t$ gives

$$\left(q_t K_t + \mathcal{B}_t - \int_{\mathcal{I}} T_t^i di\right)(1 - \varphi_t^k) = \mathcal{B}_t - \int_{\mathcal{I}} T_t^i di$$

On the other hand, the government budget constraint is

$$d\mathcal{B}_t = r_t \mathcal{B}_t dt - \left(\int_{\mathcal{I}} \tau_t^i di\right) dt$$

The government collects all payments from the households. Let

$$\tau_t \doteq \int_{\mathcal{I}} \tau_t^i di$$

Integrating the government budget constraint and imposing a no-Ponzi condition for convenience gives

$$\begin{aligned}\mathcal{B}_t &= \int_t^\infty e^{-\int_t^s r_u du}\tau_t dt = \int_t^\infty e^{-\int_t^s r_u du}\left(\int_{\mathcal{I}} \tau_t^i di\right) dt \\ &= \int_{\mathcal{I}} \left(\int_t^\infty e^{-\int_t^s r_u du}\tau_t^i dt\right) di = \int_{\mathcal{I}} T_t^i di\end{aligned}$$

where the second line swaps the order of integration (Fubini). The market value of government debt is equal to the aggregate present value of tax liabilities across all households. Hence

$$\varphi_t^k = 1$$

and the quantity of government debt $\mathcal{B}_t$ issuance has no real effect. ■

A.3 Theory Supplements

A.3.1 Government Intertemporal Budget Constraint

I next integrate the government budget constraint

$$d\mathcal{B}_t = \mathcal{B}_t r_t dt - \tau_t dt + g_t dt$$

The steps are similar to the previous derivation, with the difference that I use the market SDF ξ_t because none of the terms in the government budget constraint is driven by idiosyncratic shocks.

$$\xi_t d\mathcal{B}_t = \xi_t \mathcal{B}_t r_t dt - \xi_t \tau_t dt + \xi_t g_t dt$$

Then, using

$$\xi_t d\mathcal{B}_t = d(\mathcal{B}_t \xi_t) - \mathcal{B}_t d\xi_t$$

Gives

$$d(\mathcal{B}_t \xi_t) = \xi_t \mathcal{B}_t r_t dt - \xi_t \tau_t dt + \xi_t g_t dt + \mathcal{B}_t d\xi_t$$

Substituting the dynamics of the SDF and integrating gives

$$\begin{aligned}\int_t^\infty d(\mathcal{B}_s \xi_s) &= \int_t^\infty \xi_s \mathcal{B}_s r_s ds - \int_t^\infty \xi_s \tau_s ds + \int_t^\infty \xi_s g_s ds + \int_t^\infty \mathcal{B}_s(-\xi_s r_s ds - \pi_s \xi_s dZ_s) \\ &= \int_t^\infty \xi_t (g_s - \tau_s) ds - \int_t^\infty \mathcal{B}_s \pi_s \xi_s dZ_s\end{aligned}$$

The second integral is an Ito integral, so that

$$\lim_{T\to\infty} \mathbb{E}_t[\mathcal{B}_T \xi_T] - \mathcal{B}_t = \mathbb{E}_t\left[\int_t^\infty \xi_t (g_s - \tau_s) ds\right]$$

Rearranging, I obtain the standard government debt present value constraint (Jiang et al., 2023)

$$\mathcal{B}_t = \mathbb{E}_t\left[\int_t^\infty \xi_t(\tau_s - g_s)ds\right] + \lim_{T\to\infty} \mathbb{E}_t[\mathcal{B}_T\xi_T]$$

Lump-sum Taxation Consider now a hypothetical case where taxes are lump-sum and only driven by aggregate shocks. Suppose for simplicity that all households pay the same in each instant τ_t^l, and set $g = 0$. To be consistent with the government budget constraint, it must hold that

$$\int_{\mathcal{I}} \tau_t^l di = \tau_t$$

The market valuation of government debt is

$$\mathcal{B}_t = \mathbb{E}_t\left[\int_t^\infty \xi_t\tau_s ds\right] + \lim_{T\to\infty} \mathbb{E}_t[\mathcal{B}_T\xi_T]$$

The present value of household i's tax liabilities is then given by

$$\mathbb{E}_t\left[\int_t^\infty \xi_t^i\tau_s^l ds\right] = \mathbb{E}_t\left[\int_t^\infty \xi_t\tau_s^l ds\right]$$

Aggregating across agents and swapping order of integration gives

$$\int_{\mathcal{I}} \mathbb{E}_t\left[\int_t^\infty \xi_t\tau_s^l ds\right] di = \mathbb{E}_t\left[\int_t^\infty \xi_t\tau_s ds\right]$$

It thus follows that

$$\begin{aligned} n_t &= q_t^k k_t + \left(\mathcal{B}_t - \int_{\mathcal{I}} \mathbb{E}\left[\int_t^\infty \xi_s^i \bar{\tau}_s \tau_s^i n_t^i ds\right] di - q_t^\tau k_t\right) \\ &= q_t k_t \left(\mathbb{E}_t\left[\int_t^\infty \xi_t\tau_s ds\right] + \lim_{T\to\infty} \mathbb{E}_t[\mathcal{B}_T\xi_T] - \mathbb{E}_t\left[\int_t^\infty \xi_t\tau_s ds\right]\right) \\ &= q_t k_t + \lim_{T\to\infty} \mathbb{E}_t[\mathcal{B}_T\xi_T] \end{aligned}$$

where the second line uses $q_t^k = q_t$ since $q_t^\tau = 0$. If taxes are lump-sum, there are no service flows and the net wealth contribution of government debt is entirely due to the bubble component $\lim_{T\to\infty} \mathbb{E}_t[\mathcal{B}_T\xi_T]$. Brunnermeier et al. (2024) argue *that government debt generates a service flow from retrading even in the absence of bubbles and convenience yields.* The previous example shows that with lump-sum taxes, service flows are zero and the risk sharing benefits entirely come from the bubble component.

A.3.2 Derivation of Households' Intertemporal Budget Constraint

Let financial assets $n_t^i = q_t k_t^i + b_t^i$. The dynamic evolution of financial assets is

$$dn_t^i = r_t n_t^i dt + (dR_t^i(\iota_t) - r_t)q_t k_t^i - c_t^i dt - \tau_t^i dt$$

Recall that the individual stochastic discount factor is

$$\frac{d\xi_t^i}{\xi_t^i} = -r_t dt - \pi_t dZ_t^i$$

Multiplying the dynamic budget constraint by ξ_t^i gives

$$\xi_t^i dn_t^i = \xi_t^i r_t n_t^i dt + \xi_t^i (dR_t^i(\iota_t) - r_t dt) q_t k_t^i - \xi_t^i c_t^i dt - \xi_t^i \tau_t^i dt$$

Then, substituting

$$d(\xi_t^i n_t^i) = \xi_t^i dn_t^i + n_t^i d\xi_t^i + \langle d\xi_t^i, dn_t^i \rangle$$

gives

$$d(\xi_t^i n_t^i) = \xi_t^i r_t n_t^i dt + \xi_t^i (dR_t^i(\iota_t) - r_t dt) q_t k_t^i - \xi_t^i c_t^i dt - \xi_t^i \tau_t^i dt + n_t^i d\xi_t^i + \langle d\xi_t^i, dn_t^i \rangle$$

Rearranging the expression and collecting terms gives

$$\begin{aligned} d(\xi_t^i n_t^i) &= \xi_t^i r_t n_t^i dt + \xi_t^i (dR_t^i(\iota_t) - r_t dt) q_t k_t^i - \xi_t^i c_t^i dt - \xi_t^i \tau_t^i dt + n_t^i \left[-r_t \xi_t^i dt - \pi_t \xi_t^i dZ_t^i \right] - q_t k_t^i \nu \pi_t \xi_t^i dt \\ &= q_t k_t^i \xi_t^i \left[\mathbb{E}[dR_t^i(\iota_t)] - r_t - \nu \pi_t \right] dt - \xi_t^i c_t^i dt - \xi_t^i \tau_t^i dt - n_t^i \xi_t^i dZ_t^i + q_t k_t^i \xi_t^i \nu dZ_t^i \end{aligned}$$

Integrating from t to ∞ and taking expectations gives

$$\begin{aligned} \mathbb{E}_t \int_t^\infty d(\xi_s^i n_s^i) = \mathbb{E}_t \int_t^\infty q_s k_s^i \xi_s^i \left[\mathbb{E}[dR_s^i(\iota_s)] - r_s - \nu \pi_s \right] ds - \mathbb{E}_t \int_t^\infty \xi_s^i c_s^i dt \\ - \mathbb{E}_t \int_t^\infty \xi_s^i \tau_s^i ds - \mathbb{E}_t \int_t^\infty n_s^i \xi_s^i dZ_s^i + \mathbb{E}_t \int_t^\infty q_s k_s^i \xi_s^i \nu dZ_s^i \end{aligned}$$

The integrals with respect to dZ_t^i have expectations zero. Furthermore, because deflated returns are martingales,

$$\mathbb{E}\left[\frac{d\xi_t^i}{\xi_t^i} dR_t^i \right] = \mathbb{E}[dR_t^i(\iota_t^i)] - r_t - \pi_t \nu = 0$$

it follows that

$$\mathbb{E}_t \int_t^\infty d(\xi_s^i n_s^i) = -\mathbb{E}_t \int_t^\infty \xi_s^i c_s^i dt - \mathbb{E}_t \int_t^\infty \xi_s^i \tau_s^i ds$$

Thus

$$q_t k_t^i + b_t^i = n_t^i = \mathbb{E}_t \int_t^\infty \frac{\xi_s^i}{\xi_t^i} c_s^i dt + \mathbb{E}_t \int_t^\infty \frac{\xi_s^i}{\xi_t^i} \tau_s^i ds$$

which says that the market value of total assets equals the present discounted value of future consumption and tax liabilities. The definition

$$w_t^i = n_t^i - \mathbb{E}_t \int_t^\infty \frac{\xi_s^i}{\xi_t^i} \tau_s^i ds$$

then follows immediately.

A.4 Solving the General Model

I derive implications the full general model and derive implications for asset pricing and portfolio choice. I first show that the competitive equilibrium with taxes can be written in terms of three state variables $x_t \doteq (\mu_t, \lambda_t, \eta_t^h)$ where η_t^h is the wealth share of households with high wealth tax τ_h, and λ_t is the capital share of aggregate wealth[1]. I then characterize the dynamics of the endogenous state variables η_t^h and λ_t. Again, all proofs and omitted algebra steps are relegated to Appendices A.2 and A.3. I use a different notation than in the main text for comparison with Brunnermeier et al. (2024).

A.4.1 Equilibrium Definition

A competitive equilibrium, given *exogenous* government policy μ_t is defined in the usual way as a set of allocations and prices such that all households maximize utility and all markets clear. I take the initial capital stock k_0 and its distribution among house as given. All households are endowed with $k_t^i > 0$, so that everyone starts with strictly positive net worth.

Definition 4 (Competitive Equilibrium with Taxes)**.** *A competitive equilibrium with taxes is a set of aggregate stochastic processes: the price of capital q_t, the aggregate capital stock k_t, taxes τ_t^e, $\bar{\tau}_t$ and a set of stochastic processes for each household $i \in \mathcal{I}$: net worth n_t^i, consumption c_t^i, portfolios α_t^i, θ_t^i, investment ι_t^i such that*

(i) each household chooses c_t^i, ι_t^i, α_t^i, θ_t^i to maximize utility (20) subject to the budget constraint (30) taking aggregates as given and for arbitrary initial net worth $n_0^i > 0$;

(ii) given issuance μ_t, output taxes τ_t^e and wealth taxes $\bar{\tau}_t$ satisfy the government budget constraint (18);

(iii) aggregate capital is consistent with the initial condition k_0 and satisfies the law of motion (21);

(iv) all markets clear

$$\int_{\mathcal{I}} c_t^i di + \mathfrak{g} K_t + \int_{\mathcal{I}} \iota_t^i k_t^i di = a K_t \quad \text{(Goods)}$$

$$\int_{\mathcal{I}} \theta_t^{K,i} n_t^i di = q_t K_t \quad \text{(Capital)}$$

$$\int_{\mathcal{I}} \theta_t^i n_t^i di + \int_{\mathcal{I}} (1-\phi) \theta_t^{K,i} n_t^i di = 0 \quad \text{(Financial Assets)}$$

The market clearing condition for financial assets should be interpreted as follows. Financial assets are in zero net supply. The first term captures the total value of state-contingent contracts. The second

[1]As in the two-period model, $\lambda_t \neq 1$ means that government debt is perceived as net wealth. In several macro-finance models, it is usually the case that $\lambda_t = 1$, which follows from the fact that capital is the only source of wealth, so that $\int_{\mathcal{I}} n_t^i di = q_t k_t$. If $\lambda_t = 1$ government debt is not perceived as net wealth, but Section 3 clarifies the sense in this is informative about idiosyncratic risk-sharing.

term captures the total supply of outside equity issued by all households. The bond market clears by Walras' Law.

Brunnermeier et al. (2024) restrict their attention to a symmetric equilibrium in which all households choose the same portfolios. In my case, because of intertemporal hedging demand and heterogeneity in exposure to fiscal shocks, households may end up with levered positions in equilibrium.

A.4.2 Consumption, Investment, and Portfolio Choice

Let $x_t \doteq (\mu_t, \lambda_t, \eta_t^h)$ denote the collection of state variables. Because preferences are homothetic, I conjecture (and later verify) that households' value function takes the form

$$V(n_t^i, \tau_t^i; x_t) = \begin{cases} \frac{(n_t^i \zeta_{h,t})^{1-\gamma}}{1-\gamma} & : \quad \tau_t^i = \tau_t^h \\ \frac{(n_t^i \zeta_{l,t})^{1-\gamma}}{1-\gamma} & : \quad \tau_t^i = \tau_t^l \end{cases}$$

where $\zeta_{l,t}$ and $\zeta_{h,t}$ are functions of the aggregate state that are different across tax regimes. The functions encode forward-looking stochastic investment opportunities that each household face. These functions are determined in equilibrium and, I write

$$\frac{d\zeta_{j,t}}{\zeta_{j,t}} = \mu_{\zeta j,t} dt + \sigma_{\zeta j,t} dZ_t$$

for $j \in \{l, h\}$ and with adapted processes $\mu_{\zeta j,t} \doteq \mu_{\zeta j,t}(x_t)$ and $\sigma_{\zeta j,t} \doteq \sigma_{\zeta j,t}(x_t)$ determined in equilibrium. If $\sigma_{\zeta l,t}(x_t) \neq \sigma_{\zeta h,t}(x_t)$ there is heterogeneity in hedging behavior across households that is driven by taxation. Using the shorthand notation $V_j(n_t^i)$ to denote the value function when taxes are in regime j, the Hamilton-Jacobi-Bellman (HJB) equation associated with each household's problem if the tax rate is j

$$\begin{aligned} \max_{c_t, \iota_t, \alpha_t, \theta_t} \Bigg\{ \frac{\rho}{1-\psi} \left(\frac{c_t}{n_t}\right)^{1-\psi} \zeta_j^{\psi-1} + \mu_{nj,t} - \frac{\gamma}{2}\left(\sigma_{nj,t}^2 + \tilde{\sigma}_{nj,t}^2\right) + \mu_{\zeta j,t} \\ -\frac{\gamma}{2}\sigma_{\zeta j,t}^2 + (1-\gamma)\sigma_{nj,t}\sigma_{\zeta j,t} + \frac{\lambda_{j\to j'}}{1-\gamma}\left[\left(\frac{\zeta_{j'}}{\zeta_j}\right)^{1-\gamma} - 1\right] \Bigg\} = \frac{\rho}{1-\psi} \end{aligned} \tag{22}$$

where $\mu_{nj,t}$ captures the drift of households' net worth in state j, $\sigma_{nj,t}^2$ the loading on the aggregate shock in state j, and $\tilde{\sigma}_{nj,t}^2$ the loading on the idiosyncratic shock dW_t^i.

A few comments about the structure and notation of the HJB. The Hamilton-Jacobi-Bellman (HJB) equation is formally identical to the standard Mertonian portfolio problem, with the covariance term $\sigma_{nj,t}\sigma_{\zeta j,t}$ capturing intertemporal hedging demand. Portfolio choice does not depend directly on idiosyncratic tax or capital shocks, because the assets are only sensitive to aggregate variables. Heterogeneity in wealth taxation, however, enters the drift $\mu_{nj,t}$: households subject to higher taxes experience a faster decline in net worth ($\tau_h > \tau_l$), providing them with incentives to trade and choose different portfolios. Finally, the term on the left-hand side associated with jumps in the tax regime captures the utility gain from transitioning to regime j' and the loss from leaving regime j; this term vanishes if $\lambda_{j\to j'} = 0$. Since asset prices do not load on tax rate shocks, these jumps do not generate additional hedging demand. The first-order conditions for consumption c_t^i and investment ι_t^i then immediately imply

$$c_t^i = \rho^{1/\psi}\zeta_{j,t}^{(\psi-1)/\psi} n_t^i \tag{23}$$

$$q_t = \frac{1}{\Phi'(\iota_t)} \tag{24}$$

Because q_t is the same for everyone, all households choose the same investment rate $\iota_t^i = \iota_t$. However, the MPCs are different because of time-varying investment opportunities. In particular, high-tax rate households have higher MPCs, so that $\zeta_{h,t} > \zeta_{l,t}$ This establishes the first part of the results.

The first-order conditions hedging positions θ_t^i gives

$$\pi_t - \gamma[\chi\alpha_t^i(\sigma_{q,t} + \sigma) + \theta_t^i] + (1-\gamma)\sigma_{\zeta j,t} = 0 \tag{25}$$

Substituting this into the first-order condition for capital and plugging in $\mathbb{E}[dR_t^i]$ then gives

$$\frac{a(1-\tau_t^e) - \iota_t^i}{q_t} + \mu_{q,t} + \Phi(\iota_t^i) - \delta + \sigma_{q,t}\sigma - r_t = \pi_t(\sigma_{q,t} + \sigma) + \gamma\alpha_t^i\chi^2\nu^2 \tag{26}$$

As in standard Mertonian portfolio problems, portfolio weights are independent of individual wealth, n_t^i. Households require compensation for exposure to idiosyncratic risk, which increases with their portfolio weight α_t^i due to the skin-in-the-game constraint χ, and this compensation rises with ν. Output taxes uniformly lower the expected return on capital across all households, indicating that the mechanism operates through portfolio choice rather than investment distortions. Since all households choose the same investment ι_t^i and have identical productivity, the left-hand side of the HJB is identical for everyone regardless of their current tax regime. Consequently, exposure to idiosyncratic risk is the same across households, $\alpha_t^i = \alpha_t$, and differences in investment opportunities are absorbed entirely by hedging positions θ_t^i, rather than by differences in exposure to idiosyncratic risk. The hedging position is given by

$$\theta_t^i = \frac{\pi_t}{\gamma} - \chi\alpha_t(\sigma_{q,t} + \sigma) + \frac{1-\gamma}{\gamma}\sigma_{\zeta j,t}$$

If $\sigma_{\zeta j,t} \neq \sigma_{\zeta j',t}$, then aggregate risk is concentrated in the balance sheet of some households since

$$\sigma_{nj,t} = \frac{\pi_t}{\gamma} + \frac{1-\gamma}{\gamma}\sigma_{\zeta j,t}$$

To see how government debt impacts portfolio choice, note that from the market clearing condition

$$\int_{\mathcal{I}} \alpha_t^i n_t^i di = \alpha_t \int_{\mathcal{I}} n_t^i di = q_t k_t \Longrightarrow \alpha_t = \frac{q_t k_t}{\int_{\mathcal{I}} n_t^i di} = \lambda_t$$

where $\alpha_t = \lambda_t$ is the share of wealth invested in capital. I summarize these results in the following Lemma.

Lemma 4 (Households' Optimality). *Households' first-order conditions imply that consumption is linear in n_t^i and that the investment rate is the same for everyone $\iota_t^i = \iota_t$*

$$c_t^i = \rho^{1/\psi}\zeta_j^{(\psi-1)/\psi} n_t^i \tag{27}$$
$$q_t = \frac{1}{\Phi'(\iota_t)} \tag{28}$$

All households choose the same exposure to idiosyncratic shocks $\alpha_t^i = \alpha_t$. Expected return on capital are

$$\frac{a(1-\tau_t^e)-\iota_t^i}{q_t} + \Phi(\iota_t^i) - \delta + \mu_{q,t} + \sigma_{q,t}\sigma - r_t = \pi_t(\sigma_{q,t}+\sigma) + \underbrace{\gamma\lambda_t\chi^2\nu^2}_{\textit{id. risk premium}} \tag{29}$$

where λ_t is the wealth share of capital. Exposures to fiscal shocks are absorbed by hedging positions

$$\theta_t^i = \frac{\pi_t}{\gamma} - \chi\lambda_t(\sigma_{q,t}+\sigma) + \frac{1-\gamma}{\gamma}\sigma_{\zeta j,t}$$

If households have log utility, then $\theta_t^i = \theta_t$, $\hat{c}_t^i = \hat{c}_t$ and the equilibrium is symmetric.

The notion of wealth distribution requires particular care. Individual net worth, n_t^i, determines consumption and savings behavior and explicitly includes all untradable future tax liabilities that households owe to the government. The wealth share of capital influences the risk premium on idiosyncratic risk, although it remains to be established how the aggregate risk premium depends on λ_t; the intuition follows the two-period model, where bond interest income is discounted differently than liabilities, creating additional "resources." The idiosyncratic risk premium increases with λ_t, while its relationship with the debt-to-GDP ratio is ex-ante ambiguous. Because there is no heterogeneity in productivity, the expected return on capital is identical for all households.

Using $\tau_t^e = (1-\delta)\tau_t$ and the government budget constraint gives

$$\frac{a-\iota_t^i-g}{q_t} + \frac{\mathcal{B}_t(1-\delta)}{q_t k_t}(\mu_t - r_t) + \Phi(\iota_t^i) - \delta + \mu_{q,t} + \sigma_{q,t}\sigma - r_t = \pi_t(\sigma_{q,t}+\sigma) + \underbrace{\gamma\lambda_t\chi^2\nu^2}_{\text{id. risk premium}}$$

The average wealth tax rate is

$$\bar{\tau}_t = \frac{\mathcal{B}_t}{n_t}\frac{\delta(r_t-\mu_t)}{\tau_h\eta_t + \tau_l(1-\eta_t)} + \frac{\delta g}{\tau_h\eta_t + \tau_l(1-\eta_t)}\cdot\frac{\lambda_t}{q_t}$$

A.4.3 Proof of Lemma 4

Household i's HJB associated to his optimization problem is

$$\max_{c_t,\iota_t,\alpha_t,\theta_t}\left\{\frac{\rho}{1-\psi}\left(\frac{c_t}{n_t}\right)^{1-\psi}\zeta_j^{\psi-1} + \mu_{nj,t} - \frac{\gamma}{2}\left(\sigma_{nj,t}^2 + \tilde{\sigma}_{nj,t}^2\right) + \mu_{\zeta j,t}\right.$$
$$\left. -\frac{\gamma}{2}\sigma_{\zeta j,t}^2 + (1-\gamma)\sigma_{nj,t}\sigma_{\zeta j,t} + \frac{\lambda_{j\to j'}}{1-\gamma}\left[\left(\frac{\zeta_{j'}}{\zeta_j}\right)^{1-\gamma} - 1\right]\right\} = \frac{\rho}{1-\psi}$$

where

$$
\begin{aligned}
\frac{dn_t^i}{n_t^i} = r_t dt + \alpha_t^i(dR_t^i - r_t dt) - (1-\chi)\alpha_t^i \pi_t(\sigma_{q,t}+\sigma)dt + \theta_t^i \pi_t dt \\
- \bar{\tau}_t \tau_t^i dt - \hat{c}_t^i dt + [(\chi-1)\alpha_t^i(\sigma_{q,t}+\sigma) + \theta_t^i]dZ_t - (1-\chi)\alpha_t^i \nu dW_t^i
\end{aligned}
\tag{30}
$$

and the exposures to aggregate and idiosyncratic shocks are

$$
\begin{aligned}
\sigma_{nj,t} &= \alpha_t^i(\sigma_{q,t}+\sigma) + [(\chi-1)\alpha_t^i(\sigma_{q,t}+\sigma) + \theta_t^i] = \chi\alpha_t^i(\sigma_{q,t}+\sigma) + \theta_t^i \\
\tilde{\sigma}_{nj,t} &= \alpha_t^i \nu \chi
\end{aligned}
$$

and

$$
\frac{d\zeta_{j,t}}{\zeta_{j,t}} = \mu_{\zeta j,t} dt + \sigma_{\zeta j,t} dZ_t
$$

Recall that the return on own capital is

$$
dR_t^i = \frac{a(1-\tau_t^e) - \iota_t^i}{q_t} dt + \left(\mu_{q,t} + \Phi(\iota_t^i) - \delta + \sigma_{q,t}\sigma\right) dt + (\sigma_{q,t}+\sigma)dZ_t + \nu dW_t^i
$$

The first-order conditions for c_t^i and ι_t^i are immediate. Hence

$$
\begin{aligned}
c_t^i &= \rho^{1/\psi} \zeta_j^{(\psi-1)/\psi} n_t^i \\
q_t &= \frac{1}{\Phi'(\iota_t^i)}
\end{aligned}
$$

All households choose the same investment rate $\iota_t = \iota_t^i$. Further, consumption is linear in wealth, but it varies across tax regimes $j \in \{l, h\}$.

The first-order condition for θ_t^i is

$$
\pi_t - \gamma \left[\chi\alpha_t^i(\sigma_{q,t}+\sigma) + \theta_t^i\right] + (1-\gamma)\sigma_{\zeta j,t} = 0
$$

The first-order condition for capital holding α_t^i is slightly more involved

$$
\begin{aligned}
\mathbb{E}[dR_t^i] - r_t - (1-\chi)\pi_t(\sigma_{q,t}+\sigma) - \gamma\left[\chi\alpha_t^i(\sigma_{q,t}+\sigma) + \theta_t^i\right]\chi(\sigma_{q,t}+\sigma) \\
- \gamma\alpha_t^i(\nu\chi)^2 + (1-\gamma)\chi(\sigma_{q,t}+\sigma)\sigma_{\zeta j,t} = 0
\end{aligned}
$$

Using the first-order condition for θ_t^i, this can be rewritten as

$$
\mathbb{E}[dR_t^i] - r_t + \chi(\sigma_{q,t}+\sigma)\underbrace{\left\{\pi_t - \gamma\left[\chi\alpha_t^i(\sigma_{q,t}+\sigma) + \theta_t^i\right] + (1-\gamma)\sigma_{\zeta j,t}\right\}}_{=0} = \pi_t(\sigma_{q,t}+\sigma) + \gamma\alpha_t^i(\nu\chi)^2
$$

Hence

$$\frac{a(1-\tau_t^e)-\iota_t^i}{q_t}dt + \left(\mu_{q,t} + \Phi(\iota_t^i) - \delta + \sigma_{q,t}\sigma\right) - r_t = \pi_t(\sigma_{q,t}+\sigma) + \gamma\alpha_t^i(\nu\chi)^2$$

Because the left-hand side is the same for everyone, it follows that $\alpha_t^i = \alpha_t$ and all households choose the same exposure to their own capital. Market clearing in the capital market implies

$$\alpha_t = \lambda_t$$

Hence

$$\frac{a(1-\tau_t^e)-\iota_t^i}{q_t}dt + \left(\mu_{q,t} + \Phi(\iota_t^i) - \delta + \sigma_{q,t}\sigma\right) - r_t = \pi_t(\sigma_{q,t}+\sigma) + \gamma\lambda_t(\nu\chi)^2$$

which is the desired result. Households' equilibrium exposure to aggregate shocks equal to

$$\begin{aligned}
\sigma_{nj,t} &= \chi\alpha_t^i(\sigma_{q,t}+\sigma) + \theta_t^i \\
&= \chi\lambda_t(\sigma_{q,t}+\sigma) + \frac{1}{\gamma}\left\{\pi_t - \gamma\left[\chi\lambda_t(\sigma_{q,t}+\sigma)\right] + (1-\gamma)\sigma_{\zeta j,t}\right\} \\
&= \frac{\pi_t}{\gamma} + \frac{1-\gamma}{\gamma}\sigma_{\zeta j,t}
\end{aligned}$$

Therefore, risk is concentrated as long as $\sigma_{\zeta j,t} \neq \sigma_{\zeta j',t}$.

A.4.4 Wealth Accounting and Competitive Equilibrium

Let ξ_t^i denote household i's marginal utility process (individual SDF).[2] Integrating the dynamic budget constraint (30) and taking expectations gives the present value budget constraint

$$n_t^i = q_t^k k_t^i + \underbrace{\left(b_t^i - \mathbb{E}\left[\int_t^\infty \frac{\xi_s^i}{\xi_t^i}\bar{\tau}_s n_s^i \tau_s^i ds\right] - q_t^\tau k_t^i\right)}_{\text{net bond wealth}} + \text{Financial wealth}$$

where $b_t^i = (1-\chi\alpha_t^i - \theta_t^i)n_t^i$ is the market value of the bond position held by agent i. Here, I explicitly separate the market value of capital $q_t k_t^i$ into two components such that $q_t^k - q_t^\tau = q_t$.[3] The first term is the capitalized value of future output, while the second term is the present value of output taxes paid by household i. Financial wealth includes the market value of outside equity and hedging positions. Government debt enters household i's net worth if the share of government debt held b_t^i exceeds the present value of tax liabilities owed by household i. Aggregating and imposing market clearing in the asset markets and in the capital market gives

$$n_t = q_t^k k_t + \left(\mathcal{B}_t - \int_{\mathcal{I}} \mathbb{E}\left[\int_t^\infty \frac{\xi_s^i}{\xi_t^i}\bar{\tau}_s \tau_s^i n_s^i ds\right] di - q_t^\tau k_t\right) \tag{31}$$

[2]Provide the reference to Silva and di Tella (2020) and describe why this is the correct approach. Reference to Appendix. Make a comparison to the literature on human capital and argue that it is pretty much the same thing.

[3]In the setup of Brunnermeier et al. (2024), tax liabilities are bundled together with capital because there is only output taxation. Output taxation finances both government purchases and interest expense.

where $n_t = \int_{\mathcal{I}} n_t^i di$. Government debt is perceived as net wealth if the market value of government debt $\mathcal{B}_t$ exceeds the aggregate present value of taxes. Equation (31) however illustrates that the contribution of government debt to total wealth is not its market value, but there is a fiscal insurance component.

As in the two-period model, there are two approaches to compute the present value of aggregate tax liabilities. The first approach, which is standard, is to discount aggregate tax revenue τ_t using the market SDF ξ_t. The second approach is to compute the present value of taxes for each household, and then aggregate. If markets are complete, the two approaches coincide. If markets are incomplete and fiscal policy provides insurance, then the present value of tax liabilities is typically lower than the market value of government debt. For the calculation of aggregate wealth in equation (31), what matters is the net worth across all agents n_t^i because that is the relevant notion of wealth that determines consumption and savings decisions. Hence, to understand the impact of government debt on asset prices, the present value of taxes has to be computed by first valuing individual tax liabilities using ξ_t^i and *then* aggregating. If markets were complete, this would correspond to the present value of aggregate taxes τ_t. Here both output taxes and wealth taxes contribute to aggregate wealth because they provide insurance. Households' tax burden is higher when their capital holdings and wealth are high. This happens after a sequence of good idiosyncratic shocks. In summary, equation (31) shows that the net contribution of government debt to aggregate wealth is entirely due to the difference between the market value of debt and the individual valuations of tax claims.

Endogenous State Variables I solve for the competitive equilibrium in terms of two state variables. The first state variable is the exogenous issuance rate μ_t. The second state variable is the wealth share held by high-tax households. Let $n_t^h = \int_{\mathcal{I}_h} n_t^i di$ denote the share of wealth held by high-wealth-tax households. Define

$$\eta_t = \frac{n_t^h}{n_t^h + n_t^l}$$

The challenge with my setup is that $q_t k_t + \mathcal{B}_t \neq n_t$ because of the present value of wealth taxes. The equilibrium can be written as a function of $x_t = (\mu_t, \eta_t)$ and other model parameters. To this purpose, I follow Brunnermeier and Sannikov (2014) and specify $\Phi(\iota) = (1/\kappa)\log(1 + \kappa\iota)$. This implies $q_t = 1 + \kappa\iota_t^i$.

The strategy to solve for an equilibrium is similar to Di Tella (2017). I look for a Markov equilibrium with four state variables $x_t = (\mu_t, \eta_t)$ such that

$$\begin{gathered} q_t = q(\mu_t, \eta_t) \quad : \quad r_t = r(\mu_t, \eta_t) \quad : \quad \pi_t = \pi(\mu_t, \eta_t) \\ \zeta_{h,t} = \zeta_h(\mu_t, \eta_t) \quad : \quad \zeta_{l,t} = \zeta_l(\mu_t, \eta_t) \\ \lambda_t = \lambda_h(\mu_t, \eta_t) \quad : \quad \vartheta_t = \vartheta(\mu_t, \eta_t) \end{gathered}$$

where q_t is the equilibrium price of capital, r_t is the risk-free rate, π_t is the market price of the aggregate

Brownian risk, and $\zeta_{j,t}$, $j \in \{j, h\}$ reflects time-variation in expected returns.

A.4.5 Derivation of Households' HJB Equation – General Model

Using the shorthand $V_j(n_t^i)$, I write

$$\max_{c_t,\iota_t,\alpha_t,\theta_t} \{\mathcal{B}V_j(n_t) + f(c_t, V_j(n_t)) + \lambda_{j\to j'}\left[V_{j'}(n_t) - V_j(n_t)\right]\} = 0$$

where $\lambda_{j\to j'}$ is the jump intensity from tax regime j to tax regime j'. Substituting

$$V_j(n_t) = \frac{(n_t\zeta_j)^{1-\gamma}}{1-\gamma}$$

into the aggregator $f(c_t, V_j(n_t))$ and rearranging gives

$$\begin{aligned}
f(c_t, V_j(n_t)) &= \frac{1}{1-\psi}\left\{\frac{\rho c_t^{1-\psi}}{\left[(1-\gamma)\frac{(n_t\zeta_j)^{1-\gamma}}{1-\gamma}\right]^{(\gamma-\psi)/(1-\gamma)}} - \rho(1-\gamma)\frac{(n_t\zeta_j)^{1-\gamma}}{1-\gamma}\right\} \\
&= \frac{1}{1-\psi}\left\{\frac{\rho c_t^{1-\psi}}{(n_t\zeta_j)^{\gamma-\psi}} - \rho(n_t\zeta_j)^{1-\gamma}\right\} \\
&= \frac{\rho(n_t\zeta_j)^{1-\gamma}}{1-\psi}\left\{\frac{c_t^{1-\psi}}{(n_t\zeta_j)^{1-\psi}} - 1\right\} \\
&= \frac{\rho(n_t\zeta_j)^{1-\gamma}}{1-\psi}\left\{\left(\frac{c_t}{n_t}\right)^{1-\psi}\zeta_j^{\psi-1} - 1\right\}
\end{aligned}$$

Further, the partial derivatives of the value function can be written as

$$\begin{aligned}
V_{j,n}(n_t) &= n_t^{-\gamma}\zeta_j^{1-\gamma} = n_t^{-1}(n_t\zeta_j)^{1-\gamma} \\
V_{j,nn}(n_t) &= -\gamma n_t^{-\gamma-1}\zeta_j^{1-\gamma} = -\gamma n_t^{-2}(n_t\zeta_j)^{1-\gamma} \\
V_{j,\zeta}(n_t) &= n_t^{1-\gamma}\zeta_j^{-\gamma} = \zeta_j^{-1}(n_t\zeta_j)^{1-\gamma} \\
V_{j,\zeta\zeta}(n_t) &= -\gamma\zeta_j^{-\gamma-1}n_t^{1-\gamma} = -\gamma\zeta_j^{-2}(n_t\zeta_j)^{1-\gamma} \\
V_{j,n\zeta} &= (1-\gamma)n_t^{-\gamma}\zeta_j^{-\gamma} = (1-\gamma)n_t^{-1}\zeta_j^{-1}(n_t\zeta_j)^{1-\gamma}
\end{aligned}$$

I then write wealth evolution as

$$\frac{dn_t}{n_t} = \mu_{nj,t}dt + \sigma_{nj,t}dZ_t + \tilde{\sigma}_{nj,t}dW_t^i$$

where

$$\begin{aligned}
\sigma_{nj,t} &= [\chi\alpha_t^i(\sigma_{q,t} + \sigma) + \theta_t^i] \\
\tilde{\sigma}_{nj,t} &= \chi\alpha_t^i\nu
\end{aligned}$$

Further, since

$$\frac{d\zeta_{j,t}}{\zeta_{j,t}} = \mu_{\zeta j,t}dt + \sigma_{\zeta j,t}dZ_t$$

Implies that

$$\begin{aligned}\mathcal{B}V_j(n_t) &= V_{j,n}(n_t)n_t\mu_{nj,t} + \frac{1}{2}V_{j,nn}(n_t)n_t^2\left(\sigma_{nj,t}^2 + \tilde{\sigma}_{nj,t}^2\right) + V_{j,\zeta}(n_t)\zeta_{j,t}\mu_{\zeta j,t} \\ &\quad + \frac{1}{2}V_{j,\zeta\zeta}(n_t)\zeta_{j,t}^2\sigma_{\zeta j,t}^2 + V_{j,n\zeta}n_t\zeta_{j,t}\sigma_{nj,t}\sigma_{\zeta j,t} \\ &= (n_t\zeta_j)^{1-\gamma}\left[\mu_{nj,t} - \frac{\gamma}{2}\left(\sigma_{nj,t}^2 + \tilde{\sigma}_{nj,t}^2\right) + \mu_{\zeta j,t} - \frac{\gamma}{2}\sigma_{\zeta j,t}^2 + (1-\gamma)\sigma_{nj,t}\sigma_{\zeta j,t}\right]\end{aligned}$$

Substituting into the HJB gives

$$\begin{aligned}\max_{c_t,\iota_t,\alpha_t,\theta_t}&\Bigg\{(n_t\zeta_j)^{1-\gamma}\left[\mu_{nj,t} - \frac{\gamma}{2}\left(\sigma_{nj,t}^2 + \tilde{\sigma}_{nj,t}^2\right) + \mu_{\zeta j,t} - \frac{\gamma}{2}\sigma_{\zeta j,t}^2 + (1-\gamma)\sigma_{nj,t}\sigma_{\zeta j,t}\right] \\ &+ \frac{\rho(n_t\zeta_j)^{1-\gamma}}{1-\psi}\left\{\left(\frac{c_t}{n_t}\right)^{1-\psi}\zeta_j^{\psi-1} - 1\right\} + \lambda_{j\to j'}\left[\frac{(n_t\zeta_{j'})^{1-\gamma}}{1-\gamma} - \frac{(n_t\zeta_j)^{1-\gamma}}{1-\gamma}\right]\Bigg\} = 0\end{aligned}$$

Simplifying and rearranging finally yields equation (22) in the main text

$$\begin{aligned}\max_{c_t,\iota_t,\alpha_t,\theta_t}&\Bigg\{\frac{\rho}{1-\psi}\left(\frac{c_t}{n_t}\right)^{1-\psi}\zeta_j^{\psi-1} + \mu_{nj,t} - \frac{\gamma}{2}\left(\sigma_{nj,t}^2 + \tilde{\sigma}_{nj,t}^2\right) + \mu_{\zeta j,t} \\ &- \frac{\gamma}{2}\sigma_{\zeta j,t}^2 + (1-\gamma)\sigma_{nj,t}\sigma_{\zeta j,t} + \frac{\lambda_{j\to j'}}{1-\gamma}\left[\left(\frac{\zeta_{j'}}{\zeta_j}\right)^{1-\gamma} - 1\right]\Bigg\} = \frac{\rho}{1-\psi}\end{aligned}$$

Formal justifications for each step can be found in Achdou et al. (2021).

A.4.6 Investment Rate and Price of Capital

Substituting the optimal policies into the aggregate resource constraint gives

$$\rho^{1/\psi}\zeta_{h,t}^{(\psi-1)/\psi}n_t^h + \rho^{1/\psi}\zeta_{l,t}^{(\psi-1)/\psi}n_t^l + gk_t + \iota_t k_t = ak_t$$

Then, substituting $n_t = \frac{q_t k_t}{\lambda_t}$ and noting that $q_t = 1 + \kappa\iota_t^i$ gives

$$\frac{1+\kappa\iota_t}{\lambda_t}\left[\rho^{1/\psi}\zeta_{h,t}^{(\psi-1)/\psi}\eta_t^h + \rho^{1/\psi}\zeta_{l,t}^{(\psi-1)/\psi}(1-\eta_t^h)\right] + g + \iota_t = a$$

Solving for the investment rate ι_t gives

$$\iota_t = \frac{\lambda_t(a-g) - \rho^{1/\psi}\left[\zeta_{h,t}^{(\psi-1)/\psi}\eta_t^h + \zeta_{l,t}^{(\psi-1)/\psi}(1-\eta_t^h)\right]}{\kappa\rho^{1/\psi}\left[\zeta_{h,t}^{(\psi-1)/\psi}\eta_t^h + \zeta_{l,t}^{(\psi-1)/\psi}(1-\eta_t^h)\right] + \lambda_t}$$

from which I can immediately obtain the price of capital from the relation $q_t = 1 + \kappa \iota_t^i$

$$q_t = 1 + \kappa \frac{\lambda_t(a-g) - \rho^{1/\psi}\left[\zeta_{h,t}^{(\psi-1)/\psi}\eta_t^h + \zeta_{l,t}^{(\psi-1)/\psi}(1-\eta_t^h)\right]}{\kappa\rho^{1/\psi}\left[\zeta_{h,t}^{(\psi-1)/\psi}\eta_t^h + \zeta_{l,t}^{(\psi-1)/\psi}(1-\eta_t^h)\right] + \lambda_t}$$

From this expression, it is clear that idiosyncratic shocks do not affect the price of capital. In the special case households have log utility, $\psi = 1$, and the equilibrium simplifies to

$$\iota_t = \frac{\lambda_t(a-g) - \rho}{\kappa\rho + \lambda_t}$$

A.4.7 Derivation of Individual Stochastic Discount Factors

Let $\xi_{jt}^i = e^{-\rho t} V_{j,n}$ denote the marginal utility of wealth process. Write the local wealth evolution as

$$\frac{dn_t^i}{n_t^i} = \mu_{nj,t}dt + \sigma_{nj,t}dZ_t + \tilde{\sigma}_{nj,t}dW_t^i \tag{32}$$

Direct application of Itô's formula with jumps gives

$$d\xi_{jt}^i = -\rho e^{-\rho t}V_{j,n}dt + e^{-\rho t}V_{j,nn}dn_t^i + \frac{1}{2}e^{-\rho t}V_{j,nnn}\langle n_t^i\rangle + e^{-\rho t}\left(V_{j',n} - V_{j,n}\right)dJ_{j,t}^i$$

or

$$\begin{aligned}\frac{d\xi_{jt}^i}{\xi_{jt}^i} &= -\rho dt + \frac{V_{j,nn}}{V_{j,n}}n_t^i\mu_{nj,t}dt + \frac{1}{2}\frac{V_{j,nnn}}{V_{j,n}}(n_t^i)^2\left(\sigma_{nj,t}^2 + \tilde{\sigma}_{nj,t}^2\right)dt \\ &\quad + \frac{V_{j,nn}}{V_{j,n}}n_t^i\left(\sigma_{nj,t}dZ_t + \tilde{\sigma}_{nj,t}dW_t^i\right) + \left(\frac{V_{j',n}}{V_{j,n}} - 1\right)dJ_{j,t}^i \\ &= -\rho dt - \gamma\mu_{nj,t}dt + \frac{\gamma(\gamma+1)}{2}\left(\sigma_{nj,t}^2 + \tilde{\sigma}_{nj,t}^2\right)dt - \gamma\sigma_{nj,t}dZ_t - \gamma\tilde{\sigma}_{nj,t}dW_t^i + \left[\left(\frac{\zeta_{j',t}}{\zeta_{j,t}}\right)^{1-\gamma} - 1\right]dJ_{j,t}^i\end{aligned}$$

Then, constructing the compensated process

$$d\tilde{J}_{j,t} = -\lambda_{j\to j'}dt + dJ_{j,t}^i$$

gives

$$\begin{aligned}\frac{d\xi_{jt}^i}{\xi_{jt}^i} &= -\rho dt - \gamma\mu_{nj,t}dt + \frac{\gamma(\gamma+1)}{2}\left(\sigma_{nj,t}^2 + \tilde{\sigma}_{nj,t}^2\right)dt - \gamma\sigma_{nj,t}dZ_t \\ &\quad - \gamma\tilde{\sigma}_{nj,t}dW_t^i + \left[\left(\frac{\zeta_{j',t}}{\zeta_{j,t}}\right)^{1-\gamma} - 1\right]\left(d\tilde{J}_{j,t} + \lambda_{j\to j'}dt\right)\end{aligned}$$

As a result, using

$$r_t = -\rho - \gamma\mu_{nj,t} + \frac{\gamma(\gamma+1)}{2}\left(\sigma^2_{nj,t} + \tilde{\sigma}^2_{nj,t}\right) + \left[\left(\frac{\zeta_{j',t}}{\zeta_{j,t}}\right)^{1-\gamma} - 1\right]\lambda_{j\to j'}$$

and

$$\pi_t = \gamma\sigma_{nj,t}$$
$$\tilde{\pi}_t = -\gamma\tilde{\sigma}_{nj,t}$$

I obtain the result in the main text.

$$\frac{d\xi^i_{jt}}{\xi^i_{jt}} = -r_t dt - \pi_t dZ_t - \tilde{\pi}_t dW^i_t + \left[\left(\frac{\zeta_{j',t}}{\zeta_{j,t}}\right)^{1-\gamma} - 1\right] d\tilde{J}_{j,t}$$

A.4.8 Individual Stochastic Discount Factors, MPCs and Growth

Let $\xi^i_{jt} = e^{-\rho t}V_j(n^i_t)$. While the market SDF ξ_t only loads on the aggregate shock dZ_t, individual SDFs ξ^i_{jt} load on both the idiosyncratic capital shock and the tax shock. As a result, constructing individual SDFs is a more delicate issue because of the jump component that drives wealth tax shocks.[4] Standard arguments imply

$$\frac{d\xi^i_{jt}}{\xi^i_{jt}} = -r_t dt - \pi_t dZ_t - \tilde{\pi}_t dW^i_t + \left[\left(\frac{\zeta_{j',t}}{\zeta_{j,t}}\right)^{1-\gamma} - 1\right] d\tilde{J}_{j,t}$$

where $d\tilde{J}_{j,t}$ is a compensated jump process associated to the tax shock. The risk-free rate is

$$r_t = -\rho - \gamma\mu_{nj,t} + \frac{\gamma(\gamma+1)}{2}\left(\sigma^2_{nj,t} + \tilde{\sigma}^2_{nj,t}\right) + \left[\left(\frac{\zeta_{j',t}}{\zeta_{j,t}}\right)^{1-\gamma} - 1\right]\lambda_{j\to j'}$$

The drift r_t and the risk price π_t are common to all households, and it is easy to verify that $\mathbb{E}\left[\frac{d\xi^i_{jt}}{\xi^i_{jt}}\right] = -r_t$.[5] However, they also discount tax liabilities (both output taxes and wealth taxes) at a higher rate because individual taxation carries exposure to dW^i_t and $d\tilde{J}_{j,t}$.

If $\zeta_{h,t} > \zeta_{l,t}$, the shadow price of jump risk is positive for low tax rate households, and negative for high tax rate households. This mechanism is intuitive. A bad tax shocks means that households' tax burden increases, and the present value of their tax liabilities increases. High-tax households would like to borrow and consume a higher share of their net worth relative to low-tax households, who would rather save. What is interesting in this economy is that differences in MPCs impact the growth

[4]See Duffie (2001, Appendix I) for a formal treatment. Loosely speaking, the martingale representation property applies to the compensated process $d\tilde{J}_{j,t}$ which is constructed as

$$d\tilde{J}_{j,t} = -\lambda_{j\to j'}dt + dJ^i_{j,t}$$

This is because the counting process $dJ^i_{j,t}$ that drives the continuous time Markov chain has a predictable component, and it is not a martingale. The risk-free rate r_t subsumes the jump intensity compensation.

[5]The market SDF is the projection of each individual SDFs onto the filtration generated by Z_t, $\xi_t = \mathbb{E}_t[\xi^i_{jt}|Z_\tau, \tau \leqslant t]$.

rate of the economy, as the next Proposition shows.

Proposition 12 (Investment Rate)**.** *The equilibrium investment rate $\iota_t^i = \iota$ is*

$$\iota_t = \frac{\lambda_t(a-g) - \rho^{1/\psi}\left[\zeta_{h,t}^{(\psi-1)/\psi}\eta_t^h + \zeta_{l,t}^{(\psi-1)/\psi}(1-\eta_t^h)\right]}{\kappa\rho^{1/\psi}\left[\zeta_{h,t}^{(\psi-1)/\psi}\eta_t^h + \zeta_{l,t}^{(\psi-1)/\psi}(1-\eta_t^h)\right] + \lambda_t}$$

If $\psi = 1$, then the growth rate of the economy is independent of the wealth distribution.

The proposition shows that it is not distortions in taxation that drive investment, but rather the incentives to trade faced by heterogeneous households. The investment rate declines as the wealth share of high-tax households increases, because these households have higher marginal propensities to consume and therefore allocate a larger share of their wealth to consumption rather than investment.

A.4.9 Aggregate Risk Concentration

I need to determine whether households take different positions in the market.

Agents' wealth in different regimes accumulates at a different rate τ_j. Question: is the ratio of the two process log-normally distributed? The answer is yes.

$$d\ln(\Omega_t) = \frac{1}{\Omega_t}d\Omega_t - \frac{1}{2}\frac{1}{\Omega_t^2}\langle\Omega_t\rangle$$

where

$$\Omega_t \doteq \frac{\phi_t}{\psi_t}$$

Then

$$\frac{d\Omega_t}{\Omega_t} = (\mu_\phi - \mu_\psi + \sigma_\psi^2 - \sigma_\psi\sigma_\phi)dt + (\sigma_\phi - \sigma_\psi)dZ_t$$

So that

$$d\ln(\Omega_t) = \frac{d\Omega_t}{\Omega_t} + \frac{1}{2}\frac{\langle\Omega_t\rangle}{\Omega_t^2} = (\mu_\phi - \mu_\psi + \sigma_\psi^2 - \sigma_\psi\sigma_\phi)dt + (\sigma_\phi - \sigma_\psi)dZ_t + \frac{1}{2}(\sigma_\phi - \sigma_\psi)^2 dt$$

is normally distributed. It is then easy to solve for Ω_t is closed form. This means that the jumps size, while known at time t, it evolves following an Ito process, which is convenient to work with. Jump size is locally deterministic but it varies with aggregate shocks provided that $\sigma_\phi - \sigma_\psi \neq 0$. It is therefore very important for the richness of the theory that the two terms are different. Below, I establish that the ratio cannot be equal to one. More work needs to be done to determine under which conditions the ratio evolves with the aggregate shock.

Stochastic Investment Opportunities

The sign of the loading on dN_t depends on which regime the agent is. If the agent is in regime $j = h$, the exposure to the Brownian needs to earn a negative risk premium. If the agent is in regime $j = l$, the risk premium must be positive. From this, I can conclude that

$$\frac{\phi_t}{\psi_t} - 1 < 0 \iff \frac{\phi_t}{\psi_t} < 1$$

which implies that agents in the regime h are going to be levered and aggregate risk is concentrated within them. Thus, idiosyncratic shocks can explain how fiscal policy can lead to concentration in risk.

To verify that $\phi_t \neq \psi_t$, suppose by way of contradiction that they are the same in every period. Then all agents will choose the same portfolio weight and consumption regardless of the regime that they are in. As a result, the two HJB imply

$$r_t + \alpha_t \left(\mathbb{E}[dR_t] - r_t\right) - \phi_t^{\frac{\gamma-1}{\gamma}} \rho^{\frac{1}{\gamma}} - \tau_l - \frac{\gamma}{2}\sigma_{qt}^2 \alpha_t^2 - \frac{\rho}{1-\gamma} + \frac{\rho^{\frac{1}{\gamma}} \phi_t^{\frac{\gamma-1}{\gamma}}}{1-\gamma} + \mu_\phi - \frac{\gamma}{2}\sigma_{\phi t}^2 + (1-\gamma)\sigma_{\phi t}\sigma_{qt}\alpha_t = 0$$

and

$$r_t + \alpha_t \left(\mathbb{E}[dR_t] - r_t\right) - \psi_t^{\frac{\gamma-1}{\gamma}} \rho^{\frac{1}{\gamma}} - \tau_h - \frac{\gamma}{2}\sigma_{qt}^2 \alpha_t^2 - \frac{\rho}{1-\gamma} + \frac{\rho^{\frac{1}{\gamma}} \psi_t^{\frac{\gamma-1}{\gamma}}}{1-\gamma} + \mu_\psi - \frac{\gamma}{2}\sigma_{\psi t}^2 + (1-\gamma)\sigma_{\psi t}\sigma_{qt}\alpha_t = 0$$

subtracting the second line from the first gives

$$\tau_h - \tau_l = 0$$

which is then in contradiction to the assumption that $\tau_h > \tau_l$. Hence, the two processes are different. What is missing to establish is whether the ratio is constant or it varies over time.

Iif $\sigma_{\phi t} = \sigma_{\psi t}$, then relative investment opportunities are not affected by the aggregate shock dZ_t. If the ratio is constant, then there is no risk concentration because

$$\frac{\mathbb{E}[dR_t] - r_t}{\gamma \sigma_{qt}^2} + \frac{1-\gamma}{\gamma}\frac{\sigma_{\phi t}}{\sigma_{qt}} = \alpha_t^{(i)}$$

would then be the same for everyone. Does the ratio evolve stochastically? For that, one needs that $\sigma_{\phi t} \neq \sigma_{\psi t}$. Suppose by way of contradiction that they are the same. Then $\alpha_t^{(i)} = \alpha$ for both agents.; regardless of the taxation regime.

$$\begin{aligned} r_t + \alpha \left(\mathbb{E}[dR_t] - r_t\right) - \tau_l - \frac{\gamma}{2}\sigma_{qt}^2 \alpha^2 - \frac{\rho}{1-\gamma} + \frac{\gamma \rho^{\frac{1}{\gamma}} \phi_t^{\frac{\gamma-1}{\gamma}}}{1-\gamma} \\ + \mu_\phi - \frac{\gamma}{2}\sigma_{\phi t}^2 + (1-\gamma)\sigma_{\phi t}\sigma_{qt}\alpha + \frac{\lambda_l}{1-\gamma}\left[\left(\frac{\psi_t}{\phi_t}\right)^{1-\gamma} - 1\right] = 0 \end{aligned}$$

and

$$r_t + \alpha\left(\mathbb{E}[dR_t] - r_t\right) - \tau_h - \frac{\gamma}{2}\sigma_{qt}^2\alpha^2 - \frac{\rho}{1-\gamma} + \frac{\gamma\rho^{\frac{1}{\gamma}}\psi_t^{\frac{\gamma-1}{\gamma}}}{1-\gamma}$$
$$+ \mu_\psi - \frac{\gamma}{2}\sigma_{\psi t}^2 + (1-\gamma)\sigma_{\psi t}\sigma_{qt}\alpha_h + \frac{\lambda_h}{1-\gamma}\left[\left(\frac{\phi_t}{\psi_t}\right)^{1-\gamma} - 1\right] = 0$$

Subtracting the second line from the first line gives

$$\tau_h - \tau_l + \frac{\gamma\rho^{\frac{1}{\gamma}}}{1-\gamma}\left(\phi_t^{\frac{\gamma-1}{\gamma}} - \psi_t^{\frac{\gamma-1}{\gamma}}\right) + \mu_\phi + \frac{\lambda_l}{1-\gamma}\left[\left(\frac{\psi_t}{\phi_t}\right)^{1-\gamma} - 1\right] - \mu_\psi - \frac{\lambda_h}{1-\gamma}\left[\left(\frac{\phi_t}{\psi_t}\right)^{1-\gamma} - 1\right] = 0$$

Now, the tax rate paid by each agent in every period is $\bar{\tau}_t\tau_t^{(i)}$ where

$$\bar{\tau}_t = \frac{(r_t - \mu_{\mathcal{B}})\frac{\mathcal{B}_t}{\omega_t} + \frac{g_t}{\omega_t}}{\tau_h x_t + \tau_l(1-x_t)}$$

Thus

$$(\tau_h - \tau_l)\frac{(r_t - \mu_{\mathcal{B}})\frac{\mathcal{B}_t}{\omega_t} + \frac{g_t}{\omega_t}}{\tau_h x_t + \tau_l(1-x_t)} + \frac{\gamma\rho^{\frac{1}{\gamma}}}{1-\gamma}\left(\phi_t^{\frac{\gamma-1}{\gamma}} - \psi_t^{\frac{\gamma-1}{\gamma}}\right)$$
$$+ \mu_\phi + \frac{\lambda_l}{1-\gamma}\left[\left(\frac{\psi_t}{\phi_t}\right)^{1-\gamma} - 1\right] - \mu_\psi - \frac{\lambda_h}{1-\gamma}\left[\left(\frac{\phi_t}{\psi_t}\right)^{1-\gamma} - 1\right] = 0$$

so that the risk-free rate and g_t enters in the equation. Also $\mu_{\mathcal{B}}$ and the quantity of debt $\mathcal{B}_t$ impact that if the tax rate is heterogeneous across types. If it was not, then it would just simply drop out. If g_t loads on the Brownian, then the ratio has to depend on the Brownian as well.

A.5 Intertemporal Budget Constraints

A.5.1 Derivation of Intertemporal Budget Constraint – General Case

In the general setting, the dynamic budget constraint is

$$\frac{dn_t^i}{n_t^i} = r_t dt + \alpha_t^i(dR_t^i - r_t dt) - (1-\chi)\alpha_t^i(dR_t - r_t dt) + \theta_t^i(dR_t^\theta - r_t dt) - \bar{\tau}_t\tau_t^i dt - \hat{c}_t^i dt$$

Households' marginal utility ξ_t^i now also captures shocks to the tax regime. Ignoring outside equity and hedging positions, I obtain

$$n_t^i = \mathbb{E}\left[\int_t^\infty \xi_s^i c_s^i ds\right] = q_t k_t^i + b_t^i - \mathbb{E}\left[\int_t^\infty \xi_s^i \bar{\tau}_s \tau_s^i n_t^i ds\right]$$

where b_t^i is the market value of bond wealth, $q_t k_t^i$ is capital holdings, and $\mathbb{E}\left[\int_t^\infty \xi_s^i \bar{\tau}_s \tau_s^i ds\right]$ is the present value of wealth taxes. Importantly, the present value of capital taxes is capitalized into q_t. I then split the market value of capital into two tranches: a pure consumption stream and the present value of taxes such that

$$q_t = q_t^k - q_t^\tau$$

I can this write

$$n_t^i = (q_t^k - q_t^\tau)k_t^i + b_t^i - \mathbb{E}\left[\int_t^\infty \xi_s^i \bar{\tau}_s \tau_s^i ds\right] = q_t^k k_t^i + b_t^i - q_t^\tau k_t^i - \mathbb{E}\left[\int_t^\infty \xi_s^i \bar{\tau}_s \tau_s^i n_t^i ds\right]$$

Hence

$$n_t^i = q_t^k k_t^i + \underbrace{\left(b_t^i - \mathbb{E}\left[\int_t^\infty \xi_s^i \bar{\tau}_s \tau_s^i n_t^i ds\right] - q_t^\tau k_t^i\right)}_{\text{net bond wealth}}$$

Aggregating across households and imposing capital and asset market clearing gives

$$n_t = q_t^k k_t + \underbrace{\left(\mathcal{B}_t - \int_{\mathcal{I}} \mathbb{E}\left[\int_t^\infty \xi_s^i \bar{\tau}_s \tau_s^i n_t^i ds\right] di - q_t^\tau k_t\right)}_{\text{aggregate net bond wealth}}$$

Again, this shows that the contribution of bond wealth is positive as long as the present discounted value of tax liabilities is less than the market value of government debt. The tax claim is typically cheaper as it acts as a form of insurance. Households' tax burden increases after a sequence of good idiosyncratic shocks.

B Facts about Tax Uncertainty

B.1 List of Tax Reforms in the US

Number	Year	Reform
97-34	1981	Economic Recovery Tax Act
97-248	1982	Tax Equity and Fiscal Responsibility Act
98-21	1983	Social Security Amendments
98-369	1984	Deficit Reduction Act
99-514	1986	Tax Reform Act
100-203	1987	Omnibus Budget Reconciliation Act of 1987
101-508	1990	Omnibus Budget Reconciliation Act of 1990
103-66	1993	Omnibus Budget Reconciliation Act of 1993
105-34	1997	Taxpayer Relief Act
107-16	2001	Economic Growth and Tax Relief Reconciliation Act
107-147	2002	Job Creation and Worker Assistance Act
108-27	2003	Jobs and Growth Tax Relief Reconciliation Act
108-311	2004	Working Families Tax Relief Act
108-357	2004	American Jobs Creation Act
109-222	2005	Tax Increase Prevention and Reconciliation Act
109-280	2006	Pension Protection Act
109-432	2006	Tax Relief and Health Care Act
110-166	2007	Tax Increase Prevention Act
110-185	2008	Economic Stimulus Act
110-289	2008	Housing and Economic Recovery Act
110-343	2008	Emergency Economic Stabilization Act
111-3	2009	Children's Health Insurance Program Reauthorization Act
111-5	2009	American Recovery and Reinvestment Act
111-92	2009	Worker, Homeownership, and Business Assistance Act
111-147	2010	Hiring Incentives to Restore Employment Act
111-148; 111-152	2010	Patient Protection and Affordable Health Care Act; Health Care and Education Reconciliation Act
111-240	2010	Small Business Jobs Act
111-312	2010	Tax Relief, Unemployment Insurance Reauthorization, and Job Creation Act
112-78	2011	Temporary Payroll Tax Cut Continuation Act
112-96	2012	Middle Class Tax Relief and Job Creation Act
112-240	2012	American Taxpayer Relief Act
113-295	2014	Tax Increase Prevention Act
114-94	2015	Fixing America's Surface Transportation Act
114-113	2016	Consolidated Appropriations Act
115-97	2017	Tax Cuts and Jobs Act (original title)
116-44	2020	Further Consolidated Appropriations Act
116-127	2020	Families First Coronavirus Response Act
116-136	2020	CARES Act
116-260	2021	Consolidated Appropriations Act
117-2	2021	American Rescue Plan Act
117-169	2022	Inflation Reduction Act (original title)

B.2 Evolution of Marginal Tax Rates over Time

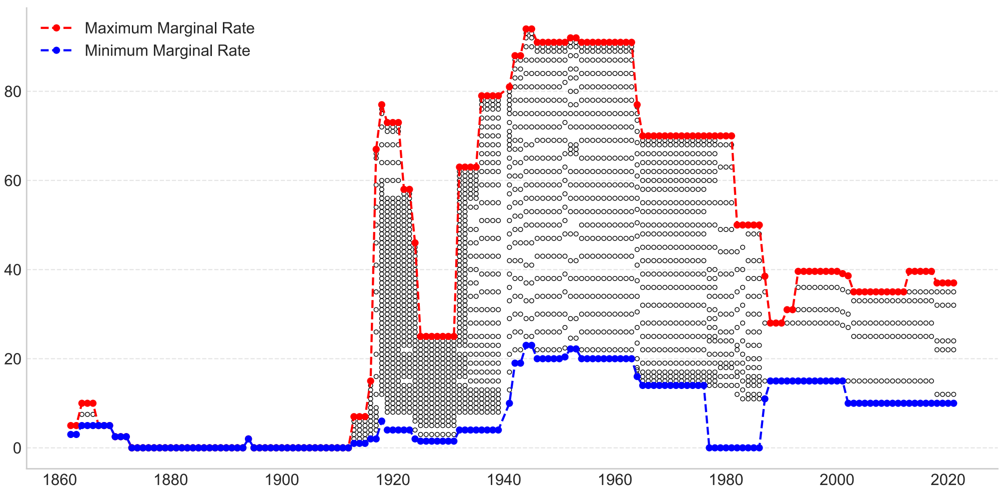


Figure 10: Evolution of marginal income tax rates over time. The red dots denote the maximum income tax rate, whereas the blue dots denote the minimum income tax rate.

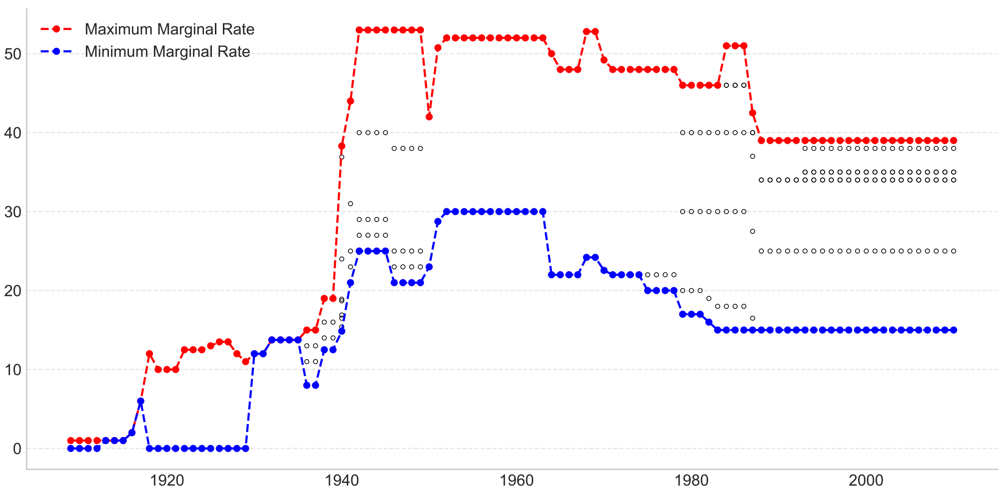
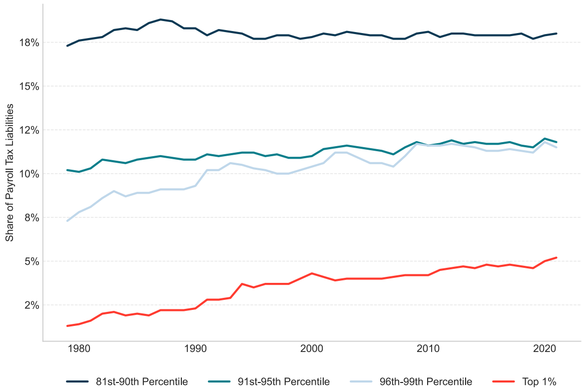
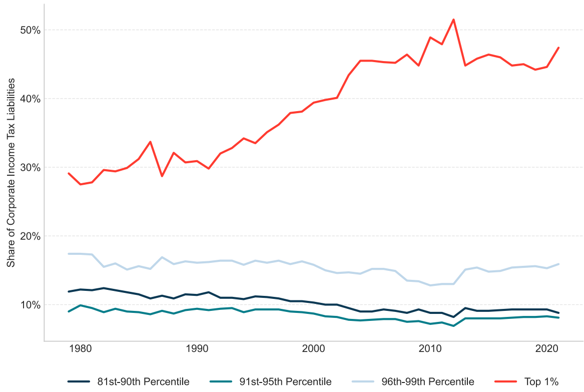


Figure 11: Evolution of marginal tax rates on corporate income over time. The red dots denote the maximum income tax rate, whereas the blue dots denote the minimum income tax rate.

B.3 Tax Burdens across the Wealth Distribution



(a) Share of payroll taxes.



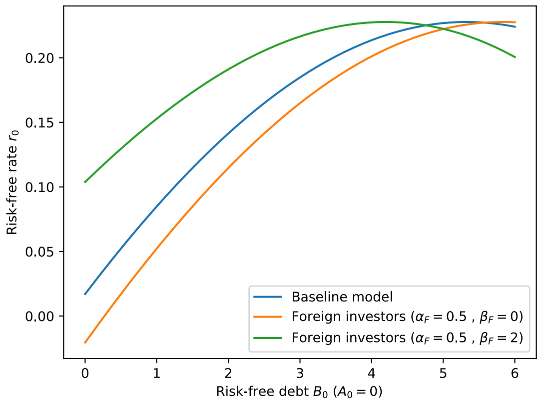
(b) Share of corporate-income taxes.

Figure 12: Share of payroll and corporate income taxes across the wealth distribution.

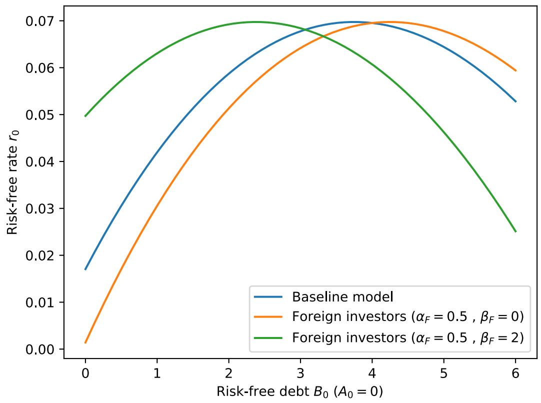
C Additional Results

C.1 Two-period Model

C.1.1 Risk-free Rate Response with Foreign Investors



(a) Output taxes ($\kappa = 0$)



(b) Output and lump-sum taxes ($\kappa = 60\%$)

Figure 13: Comparison of risk-free rate $r_0 = \frac{1}{p_0} - 1$ response to an increase in government risk-free borrowing. The left panel studies an economy in which the government only collects output taxes ($\kappa = 0$). The right panel studies an economy in which the government collects both lump-sum and output taxes ($\kappa = 60\%$). The blue line plots the risk-free rate in the benchmark economy. The orange line plots the risk-free rate when foreign investors are price-insensitive ($\alpha_F = 0.5$, $\beta_F = 0$). The green line plots the risk-free rate when foreign investors are price-sensitive ($\alpha_F = 0.5$, $\beta_F = 2$). For domestic investors, I assume CRRA utility with $\gamma = 2$ and $\beta = 0.96$.